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#7457 Summary

SUMMARY

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

This study investigates students' *commognitive* processes—the integration of communication and cognition—during problem-based learning (PBL) in mathematics. It is grounded in *commognitive theory*, which posits that thinking and communication are interdependent. An exploratory descriptive approach was applied at SMA Muhammadiyah Bangkinang, focusing on students in class XI IPA 2. Using purposive sampling, three students representing high, moderate, and low mathematical abilities were selected. Data were collected through written tests, interviews, and documentation. Researcher-developed worksheets and interview protocols, aligned with the four commognitive components—word use, visual mediators, narratives, and routines—were validated by mathematics education experts. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing. Findings indicate that high-ability students demonstrated more coherent narratives and consistent application of mathematical routines. Conversely, students with moderate and low abilities exhibited fragmented discourse and less effective use of visual mediators and terminology. The study highlights the role of commognitive factors in shaping students' mathematical discourse across ability levels. It suggests that instructional tools designed with commognitive principles can better support the development of mathematical thinking. These insights contribute to understanding the cognitive-communicative dynamics in mathematics education and inform the design of targeted learning interventions. Ethical approval was obtained, and informed consent was secured from all participants.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources

is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020)). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios.

Understanding how students engage with these materials requires a deeper analysis of their thinking processes. This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022)). According to Sfard, thinking can be viewed as an internal dialogue or self-communication.

Commognitive theory identifies four core elements of student discourse by Zayyadi, et al. (2019): (1) word use—how students employ mathematical terminology to express concepts; (2) visual mediators—such as diagrams, symbols, or images used in reasoning; (3) narratives—students' construction of mathematical statements like definitions or theorems; and (4) routines—the recognizable patterns or steps students apply when solving problems (Setyowati et al., 2022). These components are particularly important in a PBL context, as they provide a structured lens through which to analyze students' engagement with complex, real-world tasks.

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The selection of one student per ability level is justified by the case study design's focus on in-depth, context-rich exploration of individual experiences, which is a core strength of qualitative research. A sample size of three allows for a comparative analysis across diverse levels of ability while maintaining depth and thoroughness in exploring each student's experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. These instruments were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. To ensure the validity of these instruments, they were subjected to expert judgment. The validators were two mathematics education specialists and one expert in cognitive development. The rubric used for validation involved assessing the tasks' ability to provoke the four key commognitive components and whether they aligned with the research objectives. Additionally, the worksheet's clarity, mathematical relevance, and capacity to engage students were evaluated.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this approach is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

Sample questions on the worksheet aimed to assess students' mathematical reasoning in problem-solving contexts. For example, one of the problems involved a real-world scenario requiring the application of algebraic reasoning to solve a practical problem, such as determining the cost of materials given a budget constraint. Other problems involved visual aids like diagrams to assess spatial reasoning and interpretations of mathematical relationships. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

13 3. FINDINGS AND DISCUSSION

3.1 Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). The ability levels were determined using the following formulas:

Table 1. Categorization of Student Abilities Based on Scores.

Ability Category	Formula	Score Range
High	$X > M + 1SD$	$\bar{X} > 63.8$
Moderate	$M + 1SD < X \leq M - 1SD$	$35.06 \leq \bar{X} < 63.82$
Low	$X \leq M - 1SD$	$\bar{X} \leq 35.06$

3.1.1 High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time), demonstrating conceptual flexibility and fluency in mathematical discourse. This blending of technical and colloquial language suggests a strong ability to shift between abstract reasoning and practical application, which is essential for effective problem-solving. By incorporating both types of vocabulary, NSU not only shows command over formal mathematical concepts but also an awareness of how these ideas relate to real-world contexts. This ability to code-switch between different registers of language highlights a deeper understanding of mathematics beyond rote memorization.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K , P , l , L') throughout all problem-solving steps. The absence of concrete mediators was acknowledged, attributed to the unavailability of physical tools or manipulatives. This indicates not only visual literacy and symbol manipulation skills, but also adaptability in the face of material constraints. By effectively using sketches and abstract notations, NSU demonstrated an ability to visualize spatial relationships and translate them into formal representations. This strategic use of mediators reveals a capacity to shift between different semiotic systems, which is critical for advanced mathematical thinking and modeling.

Routine

NSU followed the Problem-Based Learning (PBL) steps systematically, applying appropriate formulas and calculus techniques, such as setting the derivative $L' = 0$ to find a maximum area of $5,000 \text{ m}^2$. This demonstrates a structured and goal-oriented problem-solving strategy. NSU's methodical approach reflects not only procedural fluency but also a solid conceptual understanding of optimization principles. By accurately translating the real-world context into a mathematical model and solving it analytically, NSU exhibited strong reasoning skills and the ability to navigate between abstract mathematics and practical application. This disciplined execution of PBL stages further underscores an ability to work independently and think critically within a structured framework.

Narrative

NSU accurately recalled and applied relevant information, reasoning logically through each worksheet step and verbalizing thought processes during the interview. The narrative confirms a thorough understanding of the mathematical content and reveals a high degree of metacognitive awareness. NSU consistently monitored progress, evaluated the appropriateness of strategies, and adjusted approaches when necessary—hallmarks of effective self-regulation. This reflective engagement suggests not only mastery of the material but also the ability to think critically about one's own thinking. Such metacognitive control is essential for deep learning and long-term academic success.

3.1.2 Moderate-Ability Subject (PM)

Word Use

PM used correct mathematical terms such as *area*, *side*, and *length*, indicating familiarity with foundational vocabulary. However, these terms were sometimes misaligned with the task instructions—for example, providing known information instead of independently formulating a problem. This suggests a partial understanding of the concepts and their functional use within problem-solving contexts. While PM demonstrates surface-level knowledge, the inconsistent application indicates difficulty in transferring this knowledge to new or unfamiliar tasks. The tendency to focus on recall over construction of meaning implies a need for further development in conceptual comprehension and task interpretation. Addressing this gap could support more independent and flexible problem-solving in the future.

Visual Mediators

Both iconic and symbolic mediators were used effectively—for example, sketches to represent spatial elements and symbols such as p , l , K , and L' to articulate mathematical relationships. No concrete media (e.g., manipulatives or physical models) were utilized, with time constraints cited as a limiting factor. This indicates a capacity to represent mathematical ideas visually and abstractly, suggesting visual literacy and fluency in mathematical notation. The reliance on drawings and symbols in the absence of tangible tools also demonstrates flexibility and resourcefulness under pressure. However, the lack of concrete representation might limit opportunities for hands-on exploration, which can support deeper conceptual development, especially under time constraints. Overall, the mediator choices reflect a preference for abstract and visual strategies, possibly shaped by situational demands.

Routine

PM completed all problem-solving steps, derived appropriate formulas, and correctly calculated an area of 3,200 m² using the optimization technique $L'=0L'=0L'=0$. This reflects a solid grasp of procedural steps and the mechanical application of calculus methods. However, minor conceptual slips were noted—such as imprecise interpretation of variables or partial understanding of the conditions under which maximum values occur. While PM followed the correct procedures, there appeared to be some reliance on rote methods rather than deep conceptual reasoning. This suggests that while procedural fluency is developing well, further support is needed to strengthen conceptual understanding and enhance the ability to explain *why* certain steps are taken, not just *how*.

Narrative

PM followed the task instructions but frequently placed information in incorrect sections, suggesting some confusion about the structure or expectations of the worksheet. Despite this, the use of appropriate drawings and formulas demonstrates an emerging understanding of the mathematical concepts involved. PM's attempts to represent the problem visually and symbolically indicate developing skills in mathematical modeling. These efforts reflect a growing awareness of the relationship between visual representation, algebraic formulation, and problem-solving. While structural organization needs improvement, the willingness to engage with multiple representations is a promising sign of conceptual growth and increasing mathematical confidence.

3.1.3 Low-Ability Subject (AR)

Word Use

AR used relevant mathematical terms such as *rectangle*, *length*, and *perimeter*, demonstrating familiarity with foundational geometry vocabulary. Notably, the use of dual-meaning terms like *kali* indicated attempts to bridge everyday language with mathematical ideas, revealing an intuitive but informal grasp of key concepts. However, several responses lacked precision, reflecting a limited and occasionally inconsistent use of mathematical vocabulary. This suggests that while AR possesses a basic understanding of geometric terms, there may be difficulty in selecting the most accurate or context-appropriate terminology. The blending of informal and formal language highlights the need for targeted vocabulary development to support clearer and more precise mathematical communication. Strengthening this aspect would enhance AR's ability to construct and convey mathematical reasoning more effectively.

Visual Mediators

AR used both sketches and symbolic mediators—such as P , l , L , and L' —in most steps of the problem-solving process, indicating an emerging ability to represent mathematical ideas visually and symbolically. However, there was no use of concrete visualizations (e.g., manipulatives, physical models, or real-life analogies), which may have limited deeper conceptual engagement. While the sketches and symbols suggest familiarity with abstract forms of representation, they lacked detailed elaboration or explanatory connections between visual elements and mathematical reasoning. This indicates a basic level of visual representation skill, but one that is still developing in terms of clarity, purpose, and integration with problem-solving strategies. Encouraging AR to connect visual tools more intentionally with conceptual understanding could enhance both communication and comprehension.

Routine

AR followed the worksheet structure consistently, indicating an understanding of procedural expectations and task organization. However, difficulties emerged with the correct application of formulas—particularly the use of the derivative condition $L'=0L'=0L'=0$ —which led to incorrect or incomplete conclusions. This suggests a gap in understanding the underlying principles of

optimization, such as recognizing when and why a derivative is set to zero to find a maximum or minimum. Although AR attempted to engage with formal mathematical methods, the misapplication reveals a reliance on surface-level procedures rather than conceptual reasoning. Targeted support in interpreting and applying calculus-based strategies would help AR move beyond mechanical execution toward deeper problem-solving competence.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 2. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic are used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors
Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

3.2 Discussion

3.2.2 Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical terms throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023). This precision in mathematical language aligns with the formal operational stage of Piaget's theory, where abstract and systematic thinking becomes prominent.

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023). This suggests that while PM may be transitioning into formal operational thinking, there remains reliance on concrete strategies.

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers. This supports Vygotsky's notion that language use without internalized meaning may not reflect actual cognitive development, emphasizing the need for scaffolded instruction.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p , l , and k . However, none used concrete visual media, which aligns with Piaget's formal operational stage, where individuals are expected to think abstractly without relying on tangible objects (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding. This aligns with Sfard's view of visual mediators as integral tools for the internal communication of mathematical concepts.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Although structurally accurate, their diagrams did not fully support the reasoning process, suggesting limitations in transforming visual tools into deeper understanding.

Limited-Ability Student (Subject AR)

Subject AR's visual representation was similar to the others but lacked conceptual integration, contributing to misinterpretations in the final solution. The student may have entered the formal operational stage but struggled to use visual mediators as vehicles for logical abstraction.

3.2.2 Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and maximum area via derivatives. This suggests that procedural knowledge was present across ability levels, but the depth of understanding varied.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative thinking. They modified perimeter formulas and logically sequenced steps, reflecting high procedural fluency and supporting (Mudaly & Mpofo, (2019) observation on adaptable routine use. Unlike the typical ritual routines seen in many students, NSU shifted toward personalized strategies—demonstrating a novel contribution to commognitive literature.

Moderate-Ability Student (Subject PM)

PM displayed some routine flexibility (e.g., converting formulas), but execution was inconsistent. The conceptual gaps suggest a need for guided discovery learning to foster more autonomous thinking.

Limited-Ability Student (Subject AR)

Subject AR showed formula adaptation but struggled in derivative application—indicating an incomplete grasp of higher-order routines. This highlights the need for more explicit modeling and metacognitive prompting in classroom teaching.

3.2.3 Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step using first-person pronouns, which signals metacognitive awareness (Rossydhya et al., 2021). This reflects a high level of internal dialogue, consistent with Sfard's concept of thinking as communication with the self.

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. While initial steps lacked clarity, retrospective explanations in interviews showed emerging cognitive control and reflection—suggesting growth through social interaction, in line with Vygotsky's sociocultural theory.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative formation. The final answer was incorrect, yet interview responses revealed emerging reflection skills. This underscores the potential of narrative scaffolding in enhancing self-explanation and conceptual clarity.

3.2.4 Critical Comparison with Literature

The findings affirm Setyowati et al. (2022), who reported that all commognitive components are present in students with visual learning styles. However, this study contributes novel insights by

showing that ability level influences not just the presence, but the *depth* and *accuracy* of these components. Unlike prior studies which generalized commognitive use, this research demonstrates that high-ability students engage more flexibly and metacognitively with mathematical discourse.

Furthermore, the ritualized routines discussed by (Mudaly & Mpofu, 2019) were evident across all participants, but only NSU transitioned toward exploratory and adaptive routine use—a deviation from previous findings. This supports the idea that routine development is mediated by both cognitive ability and instructional design.

3.2.5 Theoretical Contribution

This study extends Sfard's commognitive theory by applying it within the context of Indonesian problem-based learning (PBL) classrooms, offering empirical evidence on how commognitive elements manifest differently across student ability levels. Importantly, it highlights that commognition is not solely an individual process, but is shaped by instructional models, cultural norms, and teacher discourse.

By situating this analysis within a PBL framework, the study shows how structured yet open-ended learning can surface diverse depths of mathematical discourse. These findings underscore the importance of designing learning environments that allow for differentiated student expression, and encourage teachers to attend not only to student answers, but also to the *how* and *why* behind their reasoning. This has practical implications for lesson planning, formative assessment, and the integration of reflective discourse in mathematics classrooms.

4. CONCLUSION

This study found that all three students—high, medium, and low ability—showed evidence of the four components of commognitive theory (*word use, visual mediators, routines, and narratives*) while working on problem-based learning (PBL) Student Worksheets. The high-ability student used both mathematical and everyday language, applied correct formulas, used both symbolic and iconic visuals, and demonstrated strong understanding through both exploratory and ritual routines. The medium-ability student also showed all four components, but had difficulty separating the steps of problem formulation and data collection, which limited the clarity of their word use. The low-ability student demonstrated basic understanding of all four components but made errors applying derivatives, showing the need for further conceptual support. Overall, the research supports that PBL can help students of different abilities express mathematical thinking through communication and problem-solving.

However, the study had some limitations. It only involved three students, so the findings may not represent a wider group. Also, the research focused only on written worksheets and individual interviews, without examining how teacher support or peer discussions might influence students' commognitive development.

Future research should explore how changes in worksheet design or teacher guidance impact the development of each commognitive component. It would also be valuable to study how students' mathematical communication evolves over time with repeated exposure to PBL, and to observe classroom interactions to understand how teachers support student thinking. Educators are encouraged to use PBL worksheets to draw out students' mathematical reasoning and to provide extra support, especially in helping lower-ability students apply abstract concepts like derivatives more accurately.

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

This study investigates students' *commognitive* processes—the integration of communication and cognition—during problem-based learning (PBL) in mathematics. It is grounded in *commognitive theory*, which posits that thinking and communication are interdependent. An exploratory descriptive approach was applied at SMA Muhammadiyah Bangkinang, focusing on students in class XI IPA 2. Using purposive sampling, three students representing high, moderate, and low mathematical abilities were selected. Data were collected through written tests, interviews, and documentation. Researcher-developed worksheets and interview protocols, aligned with the four commognitive components—word use, visual mediators, narratives, and routines—were validated by mathematics education experts. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing. Findings indicate that high-ability students demonstrated more coherent narratives and consistent application of mathematical routines. Conversely, students with moderate and low abilities exhibited fragmented discourse and less effective use of visual mediators and terminology. The study highlights the role of commognitive factors in shaping students' mathematical discourse across ability levels. It suggests that instructional tools designed with commognitive principles can better support the development of mathematical thinking. These insights contribute to understanding the cognitive-communicative dynamics in mathematics education and inform the design of targeted learning interventions. Ethical approval was obtained, and informed consent was secured from all participants.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources

is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios.

Understanding how students engage with these materials requires a deeper analysis of their thinking processes. This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication.

Commognitive theory identifies four core elements of student discourse by Zayyadi, et al. (2019): (1) word use—how students employ mathematical terminology to express concepts; (2) visual mediators—such as diagrams, symbols, or images used in reasoning; (3) narratives—students' construction of mathematical statements like definitions or theorems; and (4) routines—the recognizable patterns or steps students apply when solving problems (Setyowati et al., 2022). These components are particularly important in a PBL context, as they provide a structured lens through which to analyze students' engagement with complex, real-world tasks.

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The selection of one student per ability level is justified by the case study design's focus on in-depth, context-rich exploration of individual experiences, which is a core strength of qualitative research. A sample size of three allows for a comparative analysis across diverse levels of ability while maintaining depth and thoroughness in exploring each student's experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. These instruments were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. To ensure the validity of these instruments, they were subjected to expert judgment. The validators were two mathematics education specialists and one expert in cognitive development. The rubric used for validation involved assessing the tasks' ability to provoke the four key commognitive components and whether they aligned with the research objectives. Additionally, the worksheet's clarity, mathematical relevance, and capacity to engage students were evaluated.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this approach is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

Sample questions on the worksheet aimed to assess students' mathematical reasoning in problem-solving contexts. For example, one of the problems involved a real-world scenario requiring the application of algebraic reasoning to solve a practical problem, such as determining the cost of materials given a budget constraint. Other problems involved visual aids like diagrams to assess spatial reasoning and interpretations of mathematical relationships. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

3.1 Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). The ability levels were determined using the following formulas:

Table 1. Categorization of Student Abilities Based on Scores.

Ability Category	Formula	Score Range
High	$X > M + 1SD$	$X > 63.8$
Moderate	$M + 1SD < X \leq M - 1SD$	$35.06 \leq X < 63.82$
Low	$X \leq M - 1SD$	$X \leq 35.06$

3.1.1 High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time), demonstrating conceptual flexibility and fluency in mathematical discourse. This blending of technical and colloquial language suggests a strong ability to shift between abstract reasoning and practical application, which is essential for effective problem-solving. By incorporating both types of vocabulary, NSU not only shows command over formal mathematical concepts but also an awareness of how these ideas relate to real-world contexts. This ability to code-switch between different registers of language highlights a deeper understanding of mathematics beyond rote memorization.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K , P , l , L') throughout all problem-solving steps. The absence of concrete mediators was acknowledged, attributed to the unavailability of physical tools or manipulatives. This indicates not only visual literacy and symbol manipulation skills, but also adaptability in the face of material constraints. By effectively using sketches and abstract notations, NSU demonstrated an ability to visualize spatial relationships and translate them into formal representations. This strategic use of mediators reveals a capacity to shift between different semiotic systems, which is critical for advanced mathematical thinking and modeling.

Routine

NSU followed the Problem-Based Learning (PBL) steps systematically, applying appropriate formulas and calculus techniques, such as setting the derivative $L' = 0$, to find a maximum area of $5,000 \text{ m}^2$. This demonstrates a structured and goal-oriented problem-solving strategy. NSU's methodical approach reflects not only procedural fluency but also a solid conceptual understanding of optimization principles. By accurately translating the real-world context into a mathematical model and solving it analytically, NSU exhibited strong reasoning skills and the ability to navigate between abstract mathematics and practical application. This disciplined execution of PBL stages further underscores an ability to work independently and think critically within a structured framework.

Narrative

NSU accurately recalled and applied relevant information, reasoning logically through each worksheet step and verbalizing thought processes during the interview. The narrative confirms a thorough understanding of the mathematical content and reveals a high degree of metacognitive awareness. NSU consistently monitored progress, evaluated the appropriateness of strategies, and adjusted approaches when necessary—hallmarks of effective self-regulation. This reflective engagement suggests not only mastery of the material but also the ability to think critically about one's own thinking. Such metacognitive control is essential for deep learning and long-term academic success.

3.1.2 Moderate-Ability Subject (PM)

Word Use

PM used correct mathematical terms such as *area*, *side*, and *length*, indicating familiarity with foundational vocabulary. However, these terms were sometimes misaligned with the task instructions—for example, providing known information instead of independently formulating a problem. This suggests a partial understanding of the concepts and their functional use within problem-solving contexts. While PM demonstrates surface-level knowledge, the inconsistent application indicates difficulty in transferring this knowledge to new or unfamiliar tasks. The tendency to focus on recall over construction of meaning implies a need for further development in conceptual comprehension and task interpretation. Addressing this gap could support more independent and flexible problem-solving in the future.

Visual Mediators

Both iconic and symbolic mediators were used effectively—for example, sketches to represent spatial elements and symbols such as p , l , K , and L' to articulate mathematical relationships. No concrete media (e.g., manipulatives or physical models) were utilized, with time constraints cited as a limiting factor. This indicates a capacity to represent mathematical ideas visually and abstractly, suggesting visual literacy and fluency in mathematical notation. The reliance on drawings and symbols in the absence of tangible tools also demonstrates flexibility and resourcefulness under pressure. However, the lack of concrete representation might limit opportunities for hands-on exploration, which can support deeper conceptual development, especially under time constraints. Overall, the mediator choices reflect a preference for abstract and visual strategies, possibly shaped by situational demands.

Routine

PM completed all problem-solving steps, derived appropriate formulas, and correctly calculated an area of 3,200 m² using the optimization technique $L'=0L'=0L'=0$. This reflects a solid grasp of procedural steps and the mechanical application of calculus methods. However, minor conceptual slips were noted—such as imprecise interpretation of variables or partial understanding of the conditions under which maximum values occur. While PM followed the correct procedures, there appeared to be some reliance on rote methods rather than deep conceptual reasoning. This suggests that while procedural fluency is developing well, further support is needed to strengthen conceptual understanding and enhance the ability to explain *why* certain steps are taken, not just *how*.

Narrative

PM followed the task instructions but frequently placed information in incorrect sections, suggesting some confusion about the structure or expectations of the worksheet. Despite this, the use of appropriate drawings and formulas demonstrates an emerging understanding of the mathematical concepts involved. PM's attempts to represent the problem visually and symbolically indicate developing skills in mathematical modeling. These efforts reflect a growing awareness of the relationship between visual representation, algebraic formulation, and problem-solving. While structural organization needs improvement, the willingness to engage with multiple representations is a promising sign of conceptual growth and increasing mathematical confidence.

3.1.3 Low-Ability Subject (AR)

Word Use

AR used relevant mathematical terms such as *rectangle*, *length*, and *perimeter*, demonstrating familiarity with foundational geometry vocabulary. Notably, the use of dual-meaning terms like *kali* indicated attempts to bridge everyday language with mathematical ideas, revealing an intuitive but informal grasp of key concepts. However, several responses lacked precision, reflecting a limited and occasionally inconsistent use of mathematical vocabulary. This suggests that while AR possesses a basic understanding of geometric terms, there may be difficulty in selecting the most accurate or context-appropriate terminology. The blending of informal and formal language highlights the need for targeted vocabulary development to support clearer and more precise mathematical communication. Strengthening this aspect would enhance AR's ability to construct and convey mathematical reasoning more effectively.

Visual Mediators

AR used both sketches and symbolic mediators—such as P , l , L , and L' —in most steps of the problem-solving process, indicating an emerging ability to represent mathematical ideas visually and symbolically. However, there was no use of concrete visualizations (e.g., manipulatives, physical models, or real-life analogies), which may have limited deeper conceptual engagement. While the sketches and symbols suggest familiarity with abstract forms of representation, they lacked detailed elaboration or explanatory connections between visual elements and mathematical reasoning. This indicates a basic level of visual representation skill, but one that is still developing in terms of clarity, purpose, and integration with problem-solving strategies. Encouraging AR to connect visual tools more intentionally with conceptual understanding could enhance both communication and comprehension.

Routine

AR followed the worksheet structure consistently, indicating an understanding of procedural expectations and task organization. However, difficulties emerged with the correct application of formulas—particularly the use of the derivative condition $L'=0L'=0L'=0$ —which led to incorrect or incomplete conclusions. This suggests a gap in understanding the underlying principles of

optimization, such as recognizing when and why a derivative is set to zero to find a maximum or minimum. Although AR attempted to engage with formal mathematical methods, the misapplication reveals a reliance on surface-level procedures rather than conceptual reasoning. Targeted support in interpreting and applying calculus-based strategies would help AR move beyond mechanical execution toward deeper problem-solving competence.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 2. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic are used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors
Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

3.2 Discussion

3.2.2 Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023). This precision in mathematical language aligns with the formal operational stage of Piaget's theory, where abstract and systematic thinking becomes prominent.

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023). This suggests that while PM may be transitioning into formal operational thinking, there remains reliance on concrete strategies.

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers. This supports Vygotsky's notion that language use without internalized meaning may not reflect actual cognitive development, emphasizing the need for scaffolded instruction.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p , l , and k . However, none used concrete visual media, which aligns with Piaget's formal operational stage, where individuals are expected to think abstractly without relying on tangible objects (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding. This aligns with Sfard's view of visual mediators as integral tools for the internal communication of mathematical concepts.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Although structurally accurate, their diagrams did not fully support the reasoning process, suggesting limitations in transforming visual tools into deeper understanding.

Limited-Ability Student (Subject AR)

Subject AR's visual representation was similar to the others but lacked conceptual integration, contributing to misinterpretations in the final solution. The student may have entered the formal operational stage but struggled to use visual mediators as vehicles for logical abstraction.

3.2.2 Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and maximum area via derivatives. This suggests that procedural knowledge was present across ability levels, but the depth of understanding varied.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative thinking. They modified perimeter formulas and logically sequenced steps, reflecting high procedural fluency and supporting (Mudaly & Mpofu, (2019) observation on adaptable routine use. Unlike the typical ritual routines seen in many students, NSU shifted toward personalized strategies—demonstrating a novel contribution to commognitive literature.

Moderate-Ability Student (Subject PM)

PM displayed some routine flexibility (e.g., converting formulas), but execution was inconsistent. The conceptual gaps suggest a need for guided discovery learning to foster more autonomous thinking.

Limited-Ability Student (Subject AR)

Subject AR showed formula adaptation but struggled in derivative application—indicating an incomplete grasp of higher-order routines. This highlights the need for more explicit modeling and metacognitive prompting in classroom teaching.

3.2.3 Narrative Construction and Cognitive Engagement**High-Ability Student (NSU Subject)**

NSU constructed a coherent narrative, personally engaging with each problem step using first-person pronouns, which signals metacognitive awareness (Rossydhya et al., 2021). This reflects a high level of internal dialogue, consistent with Sfard's concept of thinking as communication with the self.

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. While initial steps lacked clarity, retrospective explanations in interviews showed emerging cognitive control and reflection—suggesting growth through social interaction, in line with Vygotsky's sociocultural theory.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative formation. The final answer was incorrect, yet interview responses revealed emerging reflection skills. This underscores the potential of narrative scaffolding in enhancing self-explanation and conceptual clarity.

3.2.4 Critical Comparison with Literature

The findings affirm Setyowati et al. (2022), who reported that all commognitive components are present in students with visual learning styles. However, this study contributes novel insights by

showing that ability level influences not just the presence, but the *depth* and *accuracy* of these components. Unlike prior studies which generalized commognitive use, this research demonstrates that high-ability students engage more flexibly and metacognitively with mathematical discourse.

Furthermore, the ritualized routines discussed by (Mudaly & Mpofu, 2019) were evident across all participants, but only NSU transitioned toward exploratory and adaptive routine use—a deviation from previous findings. This supports the idea that routine development is mediated by both cognitive ability and instructional design.

3.2.5 Theoretical Contribution

This study extends Sfard's commognitive theory by applying it within the context of Indonesian problem-based learning (PBL) classrooms, offering empirical evidence on how commognitive elements manifest differently across student ability levels. Importantly, it highlights that commognition is not solely an individual process, but is shaped by instructional models, cultural norms, and teacher discourse.

By situating this analysis within a PBL framework, the study shows how structured yet open-ended learning can surface diverse depths of mathematical discourse. These findings underscore the importance of designing learning environments that allow for differentiated student expression, and encourage teachers to attend not only to student answers, but also to the how and why behind their reasoning. This has practical implications for lesson planning, formative assessment, and the integration of reflective discourse in mathematics classrooms.

4. CONCLUSION

This study found that all three students—high, medium, and low ability—showed evidence of the four components of commognitive theory (word use, visual mediators, routines, and narratives) while working on problem-based learning (PBL) Student Worksheets. The high-ability student used both mathematical and everyday language, applied correct formulas, used both symbolic and iconic visuals, and demonstrated strong understanding through both exploratory and ritual routines. The medium-ability student also showed all four components, but had difficulty separating the steps of problem formulation and data collection, which limited the clarity of their word use. The low-ability student demonstrated basic understanding of all four components but made errors applying derivatives, showing the need for further conceptual support. Overall, the research supports that PBL can help students of different abilities express mathematical thinking through communication and problem-solving.

However, the study had some limitations. It only involved three students, so the findings may not represent a wider group. Also, the research focused only on written worksheets and individual interviews, without examining how teacher support or peer discussions might influence students' commognitive development.

Future research should explore how changes in worksheet design or teacher guidance impact the development of each commognitive component. It would also be valuable to study how students' mathematical communication evolves over time with repeated exposure to PBL, and to observe classroom interactions to understand how teachers support student thinking. Educators are encouraged to use PBL worksheets to draw out students' mathematical reasoning and to provide extra support, especially in helping lower-ability students apply abstract concepts like derivatives more accurately.

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Dear Zulfah, Toto Nusantara, Yusuf Hanafi, Slamet Arifin

On behalf of the committee of AL-ISHLAH: Jurnal Pendidikan, we are glad to inform you that your manuscript:

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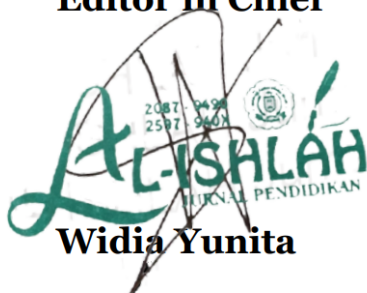
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