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AL-ISHLAH: Jurnal Pendidikan

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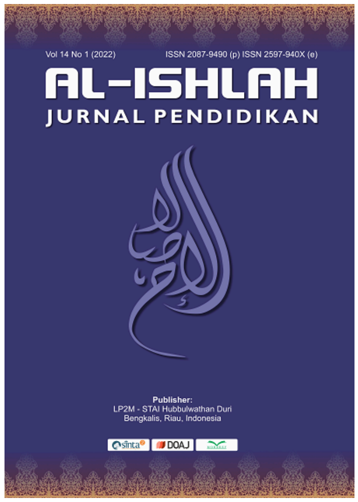

Announcements

Accreditation Status

AL-ISHLAH: Jurnal Pendidikan has applied for Reaccreditation since March 2024, and was declared to have passed the evaluation desk on May 19, 2024. Currently the journal's status is in the reaccreditation assessment period.
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Posted: 2024-12-19

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The article **publication fee** is as stated on this website and without additional costs.
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MANUSCRIPT YANG DISUBMIT DI AWAL

Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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Abstrak

This study aims to analyze students' commognitive processes in solving problem-based learning worksheets. The research employs an exploratory descriptive approach and was conducted at SMA Muhammadiyah. The research subjects were students from class XI IPA 2 at SMA Muhammadiyah Bangkinang. The sampling technique used was purposive sampling, selecting three students as research subjects: one with high ability, one with moderate ability, and one with low ability. Data were collected through written tests, interviews, and documentation. The data analysis techniques included data collection, data reduction, data presentation, and conclusion drawing.

Keywords: Analysis, Commognitive, Student Worksheets, Problem-Based Learning.

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INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources is by enhancing the quality of education (Sudarsana, 2015). Enhance and develop high-quality human resources with creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, the reality in the field differs significantly, as students are trained to memorize without receiving instructions that facilitate a comfortable learning experience (Rudiansyah et al., 2016). Another issue is that teachers consistently employ a monotonous lecture-based approach, leading to student fatigue and disengagement. (Tarigan et al., 2019). Due to these issues, teachers must utilize innovative teaching materials and instructional models to enhance the learning process. Effective and efficient student worksheets serve as essential instructional materials that should be utilized to achieve teaching and learning objectives.

Student worksheets are instructional tools that facilitate student learning and practice. This fosters an engaging collaboration between students and teachers. (Marsa et al., 2016). Student worksheets can support students in learning in a structured manner, thereby facilitating educators in creating enriched classroom experiences(Mulyani & F, 2020). Based on the aforementioned definitions, problem-based student worksheets can be considered as a model of student worksheets designed to enhance the effectiveness of learning. The development of problem-based learning student worksheets serves as a learning companion for students and is expected to improve the overall quality of education. Students' problem-solving processes can be analyzed through a commognitive approach (Presmeg, 2016).

According to Sfard, thinking is a process of communicating with oneself, either verbally or symbolically (Zayyadi & Pratiwi, 2022). Kata "*commognitive*" berasal dari kata "komunikasi" dan "kognitif", yang menunjukkan bahwa berpikir (atau berkomunikasi dengan diri sendiri) dan berkomunikasi dengan orang lain adalah hal yang sama (Sfard, 2016) . Zayyadi, et al. (2019) identifies four fundamental elements that constitute commognitive processes. First, the use of mathematical terminology through verbal expressions to represent concepts such as numbers, algebra, and equations. Second, the incorporation of visual mediators, including graphs, diagrams, and images, to facilitate mathematical reasoning. Third, the application of narratives to explain mathematical facts, encompassing axioms, definitions, formulas, and theorems. Lastly, the implementation of routines to describe the sequential steps undertaken in problem-solving processes (Setyowati et al., 2022).

Supardi et al., (2021) asserts that by analyzing commognitive processes, one can identify the challenges students face in problem-solving not only by examining the final outcomes but also by observing how words, visual mediators, narratives, and routines are utilized. In alignment with this perspective. D. Kim et al (2017) also asserts that the commognitive discourse analysis method has the potential to provide greater clarity in understanding students' mathematical thinking and the processes through which mathematics learning takes place in schools.

Commognitive analysis is conducted by researchers to examine how students communicate their knowledge in relation to Student Worksheets in Grade XI. This study contributes to education by advancing theoretical frameworks, particularly in enhancing the understanding of how cognitive processes and learning can be analyzed through problem-based learning Student Worksheets and the commognitive approach.

METODE

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. The mathematical problem sheets and interview guidelines were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. The data collection techniques employed in this study aim to systematically obtain data to address the research questions. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

RESULTS AND DISCUSSION

Hasil Penelitian

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). To categorize the measurement results into three distinct levels, the following guidelines can be utilized as a reference:

Table 1. Formulas for Categorizing Student Ability Test Instruments

Ability Category	Formula
High	$X > M + 1SD$
Moderate	$M + 1SD < X \leq M - 1SD$
Low	$X \leq M - 1SD$

Based on the application of these formulas, the range of ability scores for each student was obtained, as shown in the following table.

Table 2. Results of Student Ability Test Instrument Categorization

Ability Category	Formula
High	$X > 63.8$
Moderate	$35,06 \leq X < 63,82$
Low	$X \leq 35,06$

High-Ability Subject

Based on the student worksheet completed by the subject NSU, a commognitive analysis will be conducted on the responses in the answer sheet and the transcript of the interview with the subject. This analysis will examine the four commognitive components: word use, visual mediators, routines, and narratives.

1. Word Use

Word use, or key terminology, refers to terms used in discourse, which can include mathematical terminology as well as non-mathematical terms related to everyday life. The word use identified in the high-ability subject (NSU) can be observed in Figure 1.

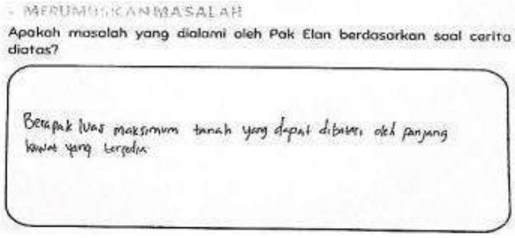
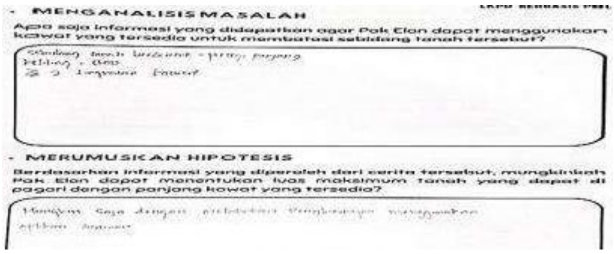


Figure 1. NSU Subject's Response Results

Based on Figure 1, it is evident that in the initial stage of problem formulation within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms area, maximum, and length. Furthermore, the word use utilized by the high-ability NSU subject in Figure 2 pertains to the stages of problem analysis and problem formulation.



Gambar 2. Hasil Jawaban Subjek NSU

Based on Figure 2, it is evident that in the problem analysis stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms rectangle and perimeter. Additionally, in the hypothesis formulation stage, the NSU subject utilizes the word use application and derivative. The word use applied by the NSU subject in the data collection stage can be observed in Figure 3.

Figure 3. NSU Subject's Response Results

Based on Figure 3, it is evident that in the data collection stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms formula and length. In the hypothesis testing stage, the word use utilized by the NSU subject can be observed in Figure 4 below.


LKPD BERBASIS PBL

Berdasarkan sketsa yang dibuat sebelumnya, apabila Pak Elan mengelilingi tanah miliknya. Tentukan berapa kali Pak Elan melewati sisi panjang dan sisi lebar?

Maka, dari jawaban anda diatas dapat kita ketahui cara mencari keliling tanah Pak Elan.

Berdasarkan penyelidikan yang telah anda lakukan pada kegiatan sebelumnya, buatlah keliling tanah Pak Elan?

Gambar 4. Hasil Jawaban Subjek NSU

Based on Figure 4, it is evident that in the data collection stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms two, one, width, and length.

The terminology used by the high-ability NSU subject in the problem-based learning (PBL) Student Worksheet can be categorized as both mathematical and non-mathematical, including terms related to everyday life. The mathematical terms identified in the NSU subject's responses include area, maximum, length, width, rectangle, one, two, derivative, mathematics, and perimeter. Meanwhile, the term that is non-mathematical or related to daily life, possessing multiple meanings, is kali.

2. Visual Mediator

The visual mediator in the responses provided by the NSU subject in the problem-based learning (PBL) Student Worksheet appears at various stages of problem-solving. In the hypothesis formulation stage, the iconic visual mediator used by the NSU subject can be observed in Figure 3, where the Student Worksheet contains depictions of land and a rectangular fence. To further interpret the meaning of these iconic visual mediators, an interview was conducted with the subject. Based on the interview results presented in Table 4, it was revealed that the NSU subject described the image as "a rectangular shape representing the land area."

Additionally, the symbolic visual mediator employed by the NSU subject is also evident in Figure 3, where, in the hypothesis formulation stage, the subject includes elements of the drawn image, such as **p** and **l**. In the data collection stage, the NSU subject writes the symbols **K, P,** and **l**. Similarly, in Figure 4, still within the data collection stage, the symbolic visual mediator consists of **K, P,** and **l**. During the hypothesis testing stage, as shown in Figure 4, the subject again employs the symbols **K, P,** and **l**. The visual mediators used by the NSU subject can be further examined in Figure 5 below.

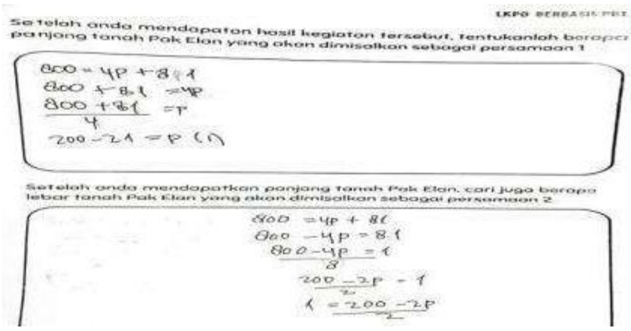


Figure 5. NSU Subject's Response Results

Based on Figure 5, it is evident that in the hypothesis testing stage, the symbolic visual mediators used by the NSU subject include K, P, and l. These symbols can also be observed in Figures 6 and 7.

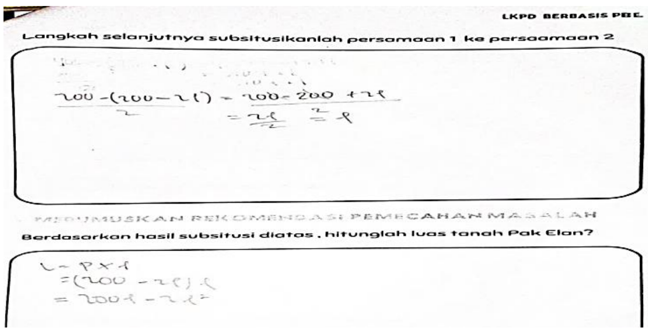


Figure 6. NSU Subject's Response Results

Based on Figure 6, the information obtained includes the use of the symbols **p** and **l**. The visual mediator used by the NSU subject can be further observed in Figure 7 below.

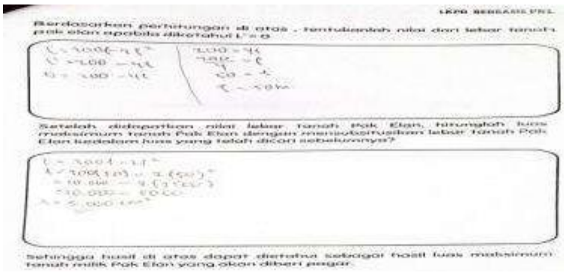


Figure 7. NSU Subject's Response Results

Based on Figure 7, the symbolic visual mediators identified include L, L’, l, and m. According to the interview with the NSU subject, the subject explained the symbols used, stating, "So the symbols present here, the first ones are K, then P, and L." However, the concrete visual mediator was not observed in the NSU subject’s responses in the Student Worksheet or the interview results. When asked about the absence of concrete visual mediators, the subject stated, "No, because there were no tools available." This indicates that the NSU subject did not employ or utilize concrete visual mediators. Based on these findings, it can be concluded that visual mediators appeared throughout various stages, including hypothesis formulation, data collection, hypothesis testing, and developing problem-solving recommendations.

3. Routine

Based on the responses written by Subject NSU in the Student Worksheets, from Image 1 to Image 7, the subject completed the PBL-based Student Worksheets by following the designated steps: formulating the problem, analyzing the problem, formulating a hypothesis, collecting data, testing the hypothesis, and formulating a recommendation for problem-solving. In Image 3, the subject wrote the formula for the perimeter of a rectangle as *P = p + 2l*, followed by the use of the area formula *A =*

p × l, and applied the derivative concept using *L' = 0*. As a result, the maximum area of Mr. Elan's land, enclosed with four levels of wire, was calculated to be 5,000 m^2.

Based on the interview results, it was found that Subject NSU explained, "I solved it by following each of the steps provided, and then created the formulas." The formulas used by Subject NSU included: "first the perimeter formula, then the area of the rectangle, and finally the formula for maximum area.

4. Narrative

To assess the mathematical understanding of Subject NSU, a high-ability student, the *narrative* component serves as a key indicator. Based on the written responses provided in the Student Worksheets, it was evident that Subject NSU accurately completed each of the required steps. In the first step, the subject successfully formulated the problem, followed by a correct analysis of the problem through identification of the relevant information provided in the worksheet. To gain deeper insight, both written responses and interview data were examined. The interview revealed that Subject NSU was able to recall and explain the given information, stating: "The information obtained from the problem includes, first, a rectangular piece of land, and second, a length of 800 meters, and umm… there will be a fence made with more than two levels." This understanding was also reflected in Subject NSU's response in Image 1.

In Image 3, Subject NSU applied the formula for the perimeter of a rectangle and, based on the scenario, chose a fence with four levels, thus multiplying the perimeter by four. In accordance with the steps outlined in the Student Worksheets, Subject NSU accurately and systematically calculated the length of the land owned by Mr. Elan. The subject also demonstrated precision in the subsequent step involving the calculation of the land's width.

Medium-Ability Subject

Based on the Student Worksheet completed by Subject PM, an analysis will be conducted to examine the commognitive components evident in the subject's responses, namely word use, visual mediators, routines, and narratives.

1. *Word Use*

Word use, or key terms, refers to expressions used in discourse, which may include both mathematical terminology and non-mathematical terms related to everyday life. The following *word use* identified in the medium-ability subject (PM) can be seen in Image 8.

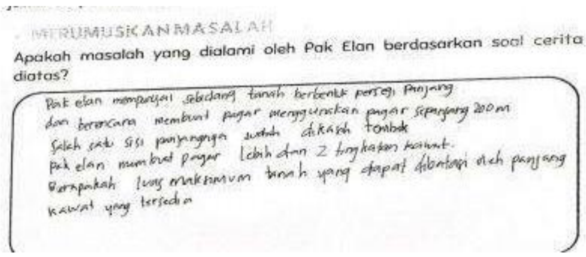


Figure 8. Results of the PM Subject Responses

The word use in the PM subject found in the responses of the Student Worksheets based on PBL can be seen in Figure 8. The information reveals that in the step of formulating the problem, the terms written by the PM subject are square, length, one, side, and area. In Figure 8, the question in the first step is about the problem statement experienced by Pak Elan; however, the subject also answered with information that appears in the question of the Student Worksheets. This indicates that the PM subject has not fully understood the intent of the first step, which is to formulate the problem.

Figure 9. Results of the PM Subject Responses

Based on Figure 9, in the step of analyzing the problem, the word use employed by the PM subject is the term "rectangle." In Figure 9, the step of analyzing the problem includes an instruction to mention all the information obtained. However, the NN subject only provided the information "K = 800 m and the land is in the shape of a rectangle" and did not mention any other information, as the remaining details were already included in the problem formulation section.

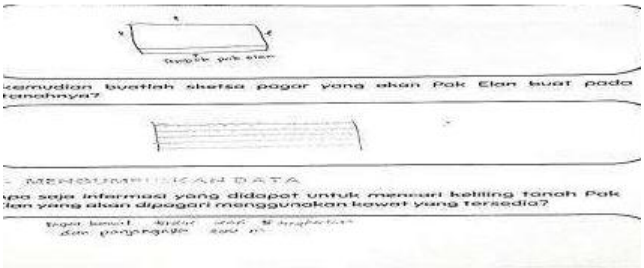
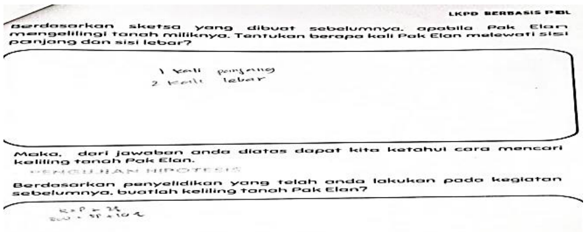


Figure 10. Results of the PM Subject Responses

Based on Figure 10, in the step of collecting data, it is evident that the word use employed by the NN subject is the term "length.



Gambar 11. Hasil Jawaban Subjek PM

Based on Figure 11, the word use found in it includes "times," "length," and "width." The terms used by the PM subject, a student with moderate ability, in the completed Student Worksheets can be categorized as either mathematical terms or terms not directly related to mathematics but connected to daily life. The terms that can be used mathematically by the PM subject include area, maximum, length, width, rectangle, times, and perimeter. On the other hand, the term "times" has dual meanings: mathematically, it refers to multiplication or calculation, while in other contexts, it can mean a river or stream (bengawan).

2. *Visual Mediator*

The visual mediator or visual mediator in the results of the Student Worksheets based on PBL for the PM subject with moderate ability, in the iconic visual mediator, can be seen in Figure 10. In the step of formulating the hypothesis, the PM subject created a drawing/sketch of Pak Elan's land and then designed a fence with five levels of wire. To understand the purpose of the image created by the PM subject, it can be found in the interview results, which state: "There is the image of Pak Elan's land that has been surrounded by a wall, and the second is a fence with five levels of wire. Meanwhile, the symbolic visual mediator found in the PM subject's response in Figure 10 is where the PM subject wrote the elements of the image, namely "p" and "l." Based on Figure 11, in the step of testing the hypothesis, the PM subject wrote the symbols "K," "P," and "l."

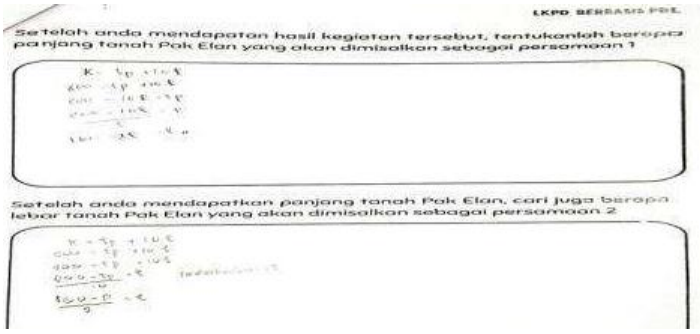


Figure 12. Results of the PM Subject Responses

Based on Figure 12, the information written by the PM subject includes the symbols "K," "p," and "l." In Figure 13, the PM subject wrote the symbols "l" and "L." The visual mediator of the PM subject can be seen in Figure 13 below.

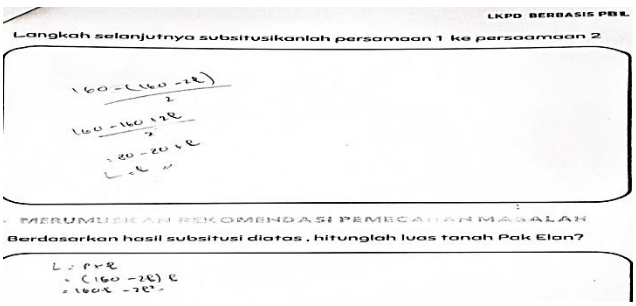


Figure 13. Results of the PM Subject Responses

Based on Figure 13, the visual mediator used by the PM subject in the stage of formulating problem-solving recommendations includes the symbols "L," "l," and "p," which are symbolic visual mediators. The visual mediator of the PM subject can be seen in Figure 14 below.

Figure 14. Results of the PM Subject Responses

Based on Figure 14, the PM subject wrote the symbol "L" as area (d), and "L'" was also found, along with the symbol for the unit "m," which represents meters. The concrete visual mediator of the PM subject is not present in the answers on the Student Worksheets based on PBL or in the interview results. The reason for this absence, according to the interview, is that the PM subject stated, "No, because I just wanted to finish quickly to catch up with time as I needed to complete other tasks." This indicates that the PM subject did not include or use any concrete visual media.

3. Routine

The routine carried out by the PM subject in the results of the Student Worksheets based on PBL, as seen in Figures 8 to 14, is the same as that of the NSU subject. The PM subject completed the Student Worksheets by following each step outlined in the worksheets. The PM subject completed the Student Worksheets based on PBL according to the steps, starting with formulating the problem, analyzing the problem, formulating the hypothesis, collecting data, testing the hypothesis, and formulating problem-solving recommendations. In Figure 9, the subject wrote the formula for the perimeter of a rectangle, which is P + 2l. The subject then used the formula for the area of a rectangle, p x l, and applied the derivative formula with L' = 0. As a result, the maximum area of Pak Elan's land with five levels of wire was obtained as 3.200 m^2

4. Narrative

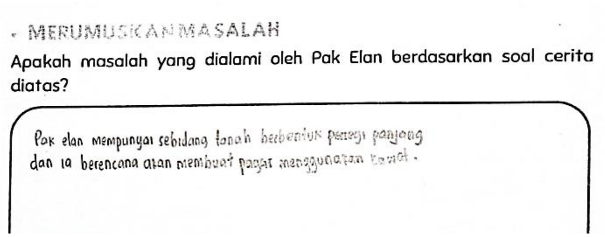
The mathematical understanding of the PM subject, a student with moderate ability, can be seen from the narratives. The NN subject completed the Student Worksheets. Based on the completed PBL-based Student Worksheets, in the problem formulation section, the NN subject provided a longer answer, but it was less accurate. In the step of analyzing the problem, the information written in the Student Worksheets shows that the PM subject only wrote "K = 800 m" and "the land is in the shape of a rectangle." The information written by the NN subject is less detailed, but the information that should have been explained in the analysis section was explained in the problem formulation section instead. Next, the PM subject followed the steps by creating a drawing. In the drawing made by the PM subject, more than two pieces of information were written, and five levels were taken into account. In the data collection section, the PM subject answered correctly. Following this, for each step in the Student Worksheets, the PM subject wrote the answers in accordance with the instructions provided.

Low Ability Subject

Based on the PBL-based Student Worksheet completed by the AR subject, a commognitive analysis will be conducted on the responses provided by the subject, focusing on the elements of word use, visual mediator, routine, and narrative.

1. Word Use

Based on Figure 15, the information in the step of formulating the problem reveals that the word use observed in the Student Worksheets responses from the AR subject is "rectangle.


- MERUMUSKAN MASALAH
Apakah masalah yang dialami oleh Pak Elan berdasarkan soal cerita diatas?

Pak elan mempunyai sebidang tanah berbentuk persegi panjang dan ia berencana akan membuat pagar menggunakan kawat .

Figure 15. Results of the PM Subject Responses

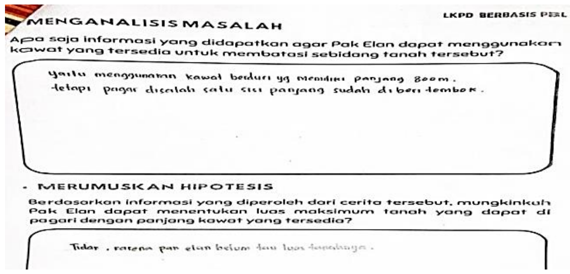


Figure 16. Results of the PM Subject Responses

Based on Figure 16, the information indicates that in the problem analysis step, the observed word use includes "length," "side," and "one.

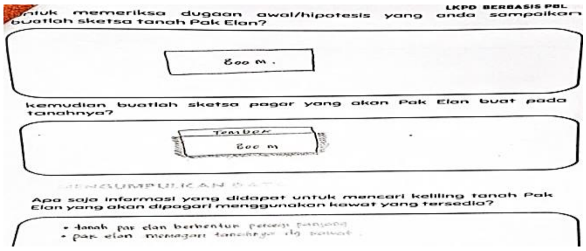


Figure 17. Results of the PM Subject Responses

Figure 18. Results of the PM Subject Responses

Based on Figure 18, the information reveals that the AR subject used the terms "times," "perimeter," "side," "length," and "width." The terms employed by the AR subject, a low-ability student, in the completed Student Worksheets can be categorized as both mathematical terms and non-mathematical terms, including those related to everyday life. The mathematical terms used by the AR subject are the same as those found in the PM subject's responses, including "area," "length," "width," "rectangle," "side," "times," and "perimeter." However, the term "times" (kali) has a dual meaning: in a mathematical context, it refers to multiplication or calculation, whereas in everyday language, it can mean "river" or "stream" (bengawan).

2. *Visual Mediator*

The visual mediator in the responses of the PBL-based Student Worksheets for the AR subject, a low-ability student, can be observed in Figure 16. In the hypothesis formulation step, the AR subject created a drawing or sketch of Pak Elan's land and then depicted a fence with four levels of wire. The purpose of the drawing made by the AR subject can be understood through the interview response: “Yes, I drew Pak Elan’s land and the fence. Meanwhile, the symbolic visual mediator found in the AR subject's response is observed in Figure 18, in the hypothesis testing step, where the AR subject wrote the symbols K, P, and l.

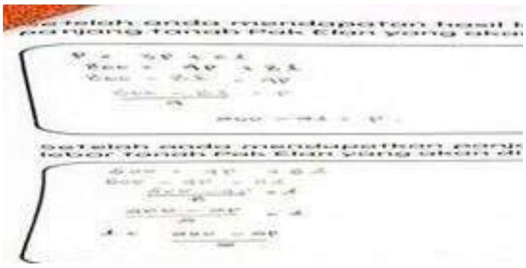


Figure 19. Results of the AR Subject Responses

Based on Figure 19, in the hypothesis testing step, the symbols written by the AR subject are **p** and **l**. Meanwhile, in Figure 20, the symbols written are the same as those in Figure 19, namely **p** and **l**, but an additional symbol **L** is also present, representing area.

Figure 20. Results of the AR Subject Responses

Based on Figure 20, in the stage of formulating and recommending a problem-solving approach, the visual mediator used by the AR subject includes the symbols l, P, and L.

Figure 21. Results of the AR Subject Responses

Based on Figure 21, the information indicates that the AR subject wrote the symbols L' and l. The concrete visual mediator of the AR subject is not present in the responses on the PBL-based Student Worksheets.

3. *Routine*

The routine followed by the AR subject in completing the PBL-based Student Worksheets, as observed in Figures 14 to 21, demonstrates that the subject adhered to each step outlined in the worksheets. The AR subject completed the Student Worksheets by following the structured steps, starting with formulating the problem, analyzing the problem, formulating a hypothesis, collecting data, testing the hypothesis, and formulating problem-solving recommendations. Based on Figure 15, in the problem formulation step, the AR subject did not provide a correct response to the question. In Figure 16, during the problem analysis step, only minimal information was provided by the AR subject. In Figure 17, the AR subject created a drawing in the hypothesis formulation step. In Figure 18, the AR subject applied the formula for the perimeter of a rectangle, P + 2l. Subsequently, the subject used the formula for the area of a rectangle, p × l. However, the AR subject was unable to correctly apply the derivative formula with L' = 0, leading to an incorrect final result.

4. *Narrative*

The mathematical understanding of the AR subject, a student with moderate ability, can be analyzed through narratives. The NN subject completed the Student Worksheets. Based on the PBL-based Student Worksheets completed by the AR subject, it is evident that in the problem formulation section, the response provided by the AR subject was incorrect. In the problem analysis step, the information recorded in the Student Worksheets shows that the AR subject only wrote "the length of the wire is 800 m and one of its sides has already been walled." The information provided by the AR subject was minimal and lacked detail. Next, the AR subject followed the structured steps by creating a drawing. In the drawing, the AR subject included more than two pieces of information, incorporating four levels. In the data collection section, the AR subject provided accurate responses. Throughout the subsequent steps in the Student Worksheets, the AR subject followed the given instructions correctly. However, despite following the procedural steps, the AR subject made an error in deriving the final result.

Diskusi

Based on the research findings on the commognitive analysis of students in solving problem-based learning (PBL) Student Worksheets, which involve the commognitive components of word use, visual mediator, narrative, and routine, data were obtained from students of Grade XI at SMA Muhammadiyah Bangkinang. The findings provide insights into how students employ these components in problem-solving.

NSU Subject – High-Ability Student

Based on the research findings, the NSU subject demonstrated all four commognitive components (word use, visual mediator, routine, and narrative). In terms of word use, the NSU subject employed both mathematical and non-mathematical terms, as observed in the Student Worksheets and interview responses. The NSU subject used seven mathematical terms and one non-mathematical term. The problem-solving stages followed by the NSU subject reflect appropriate word use, with accurate and well-structured responses provided at each step. The application of word use by the NSU subject was consistently well-executed, ensuring completeness in each stage of the problem-solving process (Zayyadi et al., 2023). The NSU subject repeatedly demonstrated word use during problem formulation, problem analysis, hypothesis formulation, data collection, and hypothesis testing.

The NSU subject demonstrated both iconic and symbolic visual mediators by drawing the shape of Pak Elan's land and fence. Within the land diagram, the NSU subject incorporated elements in the form of symbols such as p and l, along with additional symbols like K, L, L', and M. However, the NSU subject did not employ concrete visual media, stating in the interview that using additional media was unnecessary for completing the Student Worksheets. According to Jean Piaget's theory, research subjects aged 12 years and above fall within the formal operational stage, where individuals develop the ability to think abstractly and systematically (Marinda, 2020). At the formal operational stage, a key characteristic is that children no longer require concrete objects or events to aid their thinking; instead, they develop the ability to think abstractly. In terms of routine, the NSU subject applied formulas for the perimeter of a rectangle, the area of a rectangle, and the maximum area using derivative applications. The NSU subject demonstrated routine by systematically following each step in the Student Worksheets according to the instructions provided by the subject teacher. This aligns with the perspective of Mudaly & Mpofu (2019). Ritualized routines are the result of ritualized learning, where procedures are committed to memory, recalled, and mechanically applied when a specific situation requires their implementation. The NSU subject also demonstrated explorative routine, as they were able to modify the perimeter formula into p + 2l and demonstrated an understanding of its conceptual application. Mudaly & Mpofu, (2019) states that further exploration and activities should be utilized to develop these routines into a deeper understanding, allowing students to recognize various practical applications of the concept. Furthermore, routine applicability was observed in the NSU subject, as they used several symbols such as p, l, and k and also incorporated diagrams in their responses to complete the PBL-based Student Worksheets. However, routine flexibility was also evident, as the NSU subject applied multiple formulas to solve the PBL-based Student Worksheets, including the perimeter formula for a rectangle, the area formula for a rectangle, and the derivative application formula.

The narrative of the NSU subject indicates that they followed each stage outlined in the PBL-based Student Worksheets. The NSU subject also demonstrated a clear understanding of the application of each step. This is evident in the completed Student Worksheets, where the responses provided by the NSU subject are well-structured and clearly presented (Rossydha et al., 2021). The student used "I" as the object level to describe the routine they followed in completing the tasks. This indicates a personal engagement with the problem-solving process, demonstrating an awareness of the steps taken and the reasoning behind them.

It can be concluded that the NSU subject, in the responses provided in the problem-based learning (PBL) Student Worksheets on the topic of derivative applications, demonstrated all four commognitive components. Within the Student Worksheets, the NSU subject systematically and accurately completed each section, following the steps correctly and appropriately. This finding aligns with research conducted by Setyowati et al., (2022) highlighted the emergence of all four commognitive components in students with a visual learning style. Similarly, the NSU subject has demonstrated the ability to effectively communicate their thought processes, both internally and in interactions with others (Rossydha et al., 2021). Through the commognitive framework, the NSU subject has demonstrated the ability to communicate their thoughts in a structured and logical manner.(Setyowati et al., 2022).

Subject PM – Students with Moderate Ability

Based on the research findings, it was found that Subject PM demonstrated all four commognitive components—word use, visual mediators, routines, and narratives. In terms of word use, Subject PM employed both mathematical and non-mathematical terms, as evident in the Student Worksheets and interview responses. The word use observed in Subject PM appeared predominantly during the problem comprehension stage (Zayyadi et al., 2023). However, Subject PM demonstrated inaccuracies in completing the steps outlined in the Student Worksheets, particularly in the sections concerning problem formulation and problem analysis.

Visual mediators function as tools for presenting problems through visual representations, including images or diagrams (Setyowati et al., 2022). Subject PM utilized both iconic and symbolic visual mediators by drawing the shape of the land and the fence. Within the land illustration, the subject included the symbols p and q to represent specific elements of the problem. Similar to Subject NSU, Subject PM did not use any concrete visual media, stating that the use of additional materials was unnecessary for completing the Student WorksheetsAccording to Jean Piaget's theory as cited in Marinda, (2020) research subjects aged 12 years and above are generally in the formal operational stage. This stage is characterized by the individual's ability to think without relying on concrete objects or events. During this developmental period, children are capable of abstract reasoning and can mentally manipulate ideas and hypothetical situations.

In the aspect of routine, Subject PM applied formulas for the perimeter and area of a rectangle, as well as the formula for maximum area found in the application of derivatives. This indicates that Subject PM demonstrated routine flexibility, as evidenced by the use of more than one method or formula to solve the problem. Additionally, Subject PM employed ritual routines by following step-by-step procedures outlined in the Student Worksheets and adhering to the instructional methods previously taught by the subject-matter teacher (Mudaly & Mpofu, 2019). Exploratory routines and extended activities should be employed to further develop these routines into deeper conceptual understanding, enabling learners to recognize various practical applications of the concept (Mudaly & Mpofu, 2019). In this context, the exploratory routine demonstrated by Subject PM was similar to that of Subject NSU, as both were able to modify the formula for the perimeter of a rectangle. Subject PM was also able to determine calculations based on a fencing model involving five levels of wire. Furthermore, routine applicability was evident in Subject PM's work, as the subject utilized several symbols such as p, l, and k, and included a drawing in the response sheet to complete the PBL-based Student Worksheet.

The narrative of Subject PM was similar to that of Subject NSU in that Subject PM did not fully follow each step outlined in the Student Worksheets. An examination of the completed Student Worksheets revealed that Subject PM's responses in the problem formulation and problem analysis sections were not entirely accurate. However, during the interview, Subject PM was able to articulate and demonstrate an understanding of the tasks completed. In the subsequent steps, Subject PM showed a clear understanding of how to apply each stage in solving the problem, including the correct use of procedures and formulas, as confirmed through interview data. As a result, the final version of the Student Worksheets completed by the subject appeared more systematic.

As a result, the final version of the Student Worksheets completed by the subject appeared more systematic. It can therefore be concluded that Subject PM, in the completed Student Worksheets, demonstrated all four components of commognitive theory, in line with the findings reported by Setyowati et al., (2022) identified all four components of commognitive theory within the context of visual learning styles. In the case of Subject PM, the Student Worksheets were completed fairly well; however, there was a lack of full understanding and accuracy in one particular step—namely, the formulation and analysis of the problem. Based on the interview results, Subject PM demonstrated the ability to analyze the problem effectively, suggesting that the subject's interpersonal communication was stronger than their intrapersonal communication.

Subject AR: Learners with Limited Academic Proficiency

Based on the research findings, Subject AR demonstrated all four components of commognitive theory: word use, visual mediators, routines, and narratives. In terms of word use, Subject AR employed both mathematical and non-mathematical terminology, which appeared in the Student Worksheets as well as in the interview responses. The use of mathematical language by Subject AR began at the problem formulation stage, during which the subject used seven mathematical terms and one non-mathematical term.

Subject AR employed both iconic and symbolic visual mediators by drawing the shape of the land and the surrounding fence. Similar to Subjects PM and NSU, Subject AR did not utilize any concrete visual media, stating that there was no need to use additional materials, as drawing directly in the Student Worksheets was considered sufficient.

Subject AR applied formulas for the perimeter and area of a rectangle, as well as the formula for maximum area within the context of derivative applications. This indicates that Subject AR demonstrated routine flexibility, as evidenced by the use of multiple methods or formulas to solve the problem. In addition, Subject AR exhibited ritual routines by following the procedural steps outlined in the Student Worksheets and those taught by the subject-matter teacher. Exploratory routines were also evident, as Subject AR was able to develop and modify several formulas, including the formula for the perimeter of a rectangle. Mudaly & Mpofu, (2019) stated that exploration and extended activities should be used to develop routines into deeper conceptual understanding, enabling learners to recognize various practical applications of the concept. In line with this, routine applicability was evident in Subject AR's work, as the subject used several symbols such as p, l, and k, and included a drawing in the response sheet to complete the PBL-based Student Worksheet.

Subject AR followed the steps outlined in the Student Worksheets; however, the subject did not fully understand the use of each step and the application of relevant procedures. This was evident from the completed Student Worksheets, particularly in the problem formulation step, where the response was inaccurate, and in the problem analysis section, where only limited information was provided.

Nevertheless, based on the interview results, Subject AR was able to articulate relevant information and demonstrate an ability to analyze the problem. In the final step of the Student Worksheets, Subject AR's answer was incorrect due to a misunderstanding of the concept of derivative applications. However, the subject showed an adequate understanding of the formulas used, indicating a partial grasp of their conceptual application.

It can therefore be concluded that Subject AR demonstrated all four components of commognitive theory in the completed Student Worksheets, consistent with the findings reported in the study conducted by Setyowati et al., (2022) identified all four components of commognitive theory in relation to visual and kinesthetic learning styles. Based on the completed Student Worksheets, Subject AR showed some similarities with Subject PM, particularly in demonstrating a relatively good level of completion. Furthermore, interview data indicated that Subject AR was capable of analyzing the problem, suggesting that both intrapersonal and interpersonal communication skills were adequately developed. In the final step, however, Subject AR provided an incorrect final answer in the Student Worksheets. Nevertheless, the subject demonstrated a reasonable understanding of the procedural steps and the application of formulas required to complete the task.

CONCLUSION

Based on the presentation of research findings and discussion, it can be concluded that the high-ability subject demonstrated all four components of commognitive theory within the context of the problem-based learning Student Worksheets. The subject employed *word use* that included both mathematical and non-mathematical terms. The *visual mediators* used were both iconic and symbolic in nature. In terms of *routines*, the subject applied formulas related to the area and perimeter of rectangles, as well as the formula for maximum area in the context of derivatives. The subject also engaged in both *ritual* and *exploratory* routines. The *narrative* component was reflected in the subject's demonstrated understanding of how to complete the Student Worksheets and apply the appropriate procedures and formulas.

The medium-ability subject demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. However, in the step of problem formulation, the subject included some information that overlapped with the data collection phase, resulting in limited written input during the data-gathering section. Consequently, the *word use* component appeared only to a minimal extent. The subject used both mathematical and non-mathematical terms. The *visual mediators* employed by the medium-ability subject were similar to those used by the high-ability subject, consisting of both iconic and symbolic representations. Regarding *routines*, the subject applied formulas for the area and perimeter of rectangles, as well as for determining the maximum area using derivatives. Similar to the high-ability subject, the medium-ability subject also demonstrated both *ritual* and *exploratory* routines. The *narrative* component was reflected in the subject’s understanding of how to complete the problem-based learning Student Worksheets and apply the relevant procedures and formulas.

The low-ability subject also demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. The subject used both mathematical and non-mathematical terminology as part of the *word use* component. The *visual mediators* employed were iconic and symbolic, similar to those used by the high-ability subject. In terms of *routines*, the subject engaged in both *ritual* and *exploratory* routines. However, the subject provided an incorrect answer in the final step due to a misunderstanding of the concept of derivative applications. The *narrative* component was evident in the subject’s understanding of several procedural steps within the Student Worksheets and a partial grasp of the formulas and procedures used.

It can therefore be concluded that all three subjects—those with high, medium, and low ability—demonstrated the four components of commognitive theory in completing the problem-based learning Student Worksheets. The use of problem-based learning worksheets had a positive impact on students' communication processes and their thinking in relation to problem-solving.

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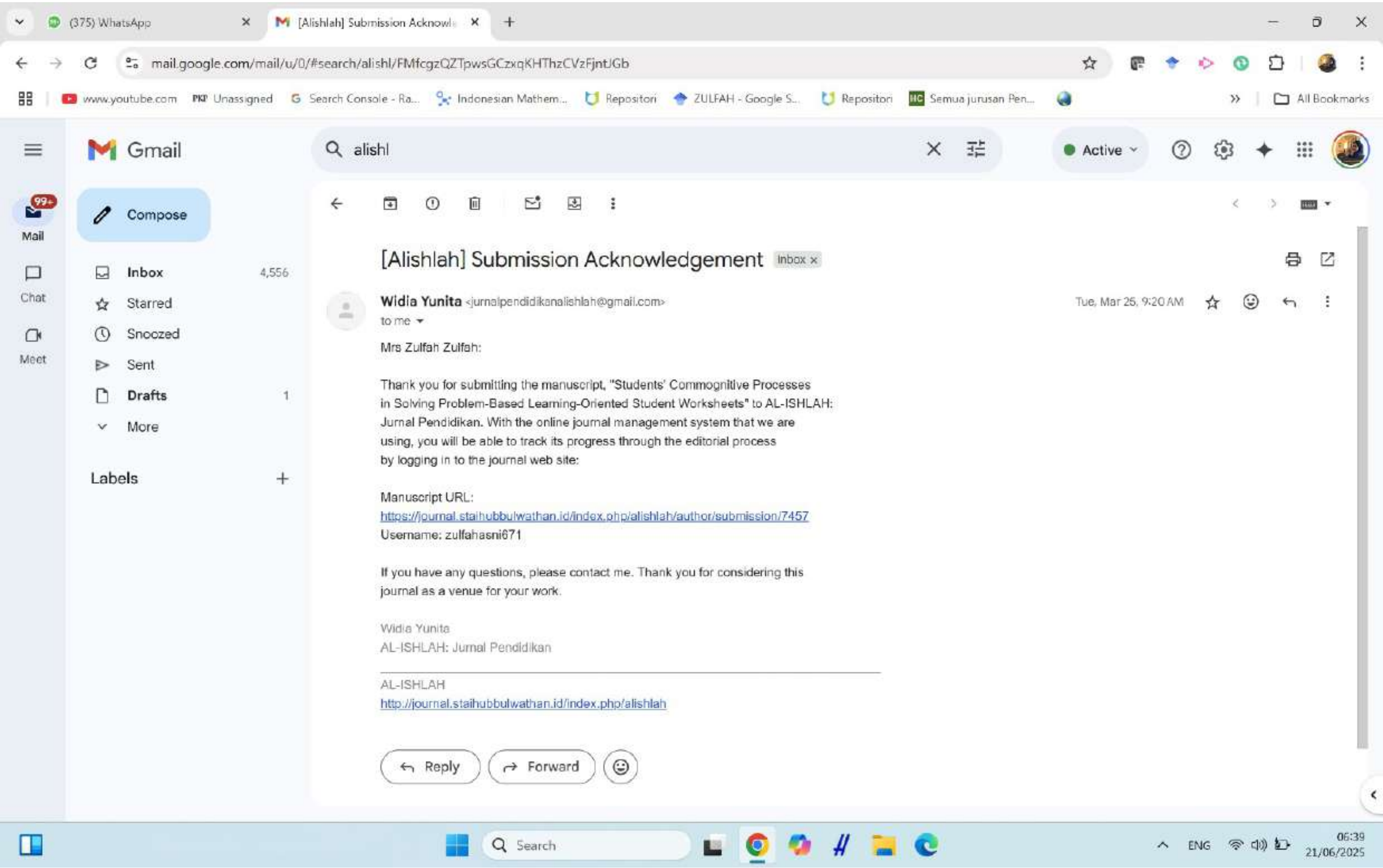
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#7457 Summary

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Title and Abstract

Title: Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

Abstract: This study investigates students' commognitive processes—the integration of communication and cognition—during problem-based learning (PBL) in mathematics. It is grounded in *commognitive theory*, which posits that thinking and communication are interdependent. An exploratory descriptive approach was applied at SMA Muhammadiyah Bangkinang, focusing on students in class XI IPA 2. Using purposive sampling, three students representing high, moderate, and low mathematical abilities were selected. Data were collected through written tests, interviews, and documentation. Researcher-developed worksheets and interview protocols, aligned with the four commognitive components—word use, visual mediators, narratives, and routines—were validated by mathematics education experts. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing.
Findings indicate that high-ability students demonstrated more coherent narratives and consistent application of mathematical routines. Conversely, students with moderate and low abilities exhibited fragmented discourse and less effective use of visual mediators and terminology. The study highlights the role of commognitive factors in shaping students' mathematical discourse across ability levels. It suggests that instructional tools designed with commognitive principles can better support the development of mathematical thinking. These insights contribute to understanding the cognitive-communicative dynamics in mathematics education and inform the design of targeted learning interventions. Ethical approval was obtained, and informed consent was secured from all participants.

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Zulfah Zulfah

We have reached a decision regarding your submission to AL-ISHLAH: Jurnal Pendidikan, "Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets ".

Our decision is: Revisions Required

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Versi Reviewer 1

Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

Abstrak

This study aims to analyze students' commognitive processes in solving problem-based learning worksheets. The research employs an exploratory descriptive approach and was conducted at SMA Muhammadiyah. The research subjects were students from class XI IPA 2 at SMA Muhammadiyah Bangkinang. The sampling technique used was purposive sampling, selecting three students as research subjects: one with high ability, one with moderate ability, and one with low ability. Data were collected through written tests, interviews, and documentation. The data analysis techniques included data collection, data reduction, data presentation, and conclusion drawing.

Keywords: Analysis, Commognitive, Student Worksheets, Problem-Based Learning.

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INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources is by enhancing the quality of education (Sudarsana, 2015). Enhance and develop high-quality human resources with creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, the reality in the field differs significantly, as students are trained to memorize without receiving instructions that facilitate a comfortable learning experience (Rudiansyah et al., 2016). Another issue is that teachers consistently employ a monotonous lecture-based approach, leading to student fatigue and disengagement. (Tarigan et al., 2019). Due to these issues, teachers must utilize innovative teaching materials and instructional models to enhance the learning process. Effective and efficient student worksheets serve as essential instructional materials that should be utilized to achieve teaching and learning objectives.

Commented [H.1]: • Briefly mention what "commognitive" refers to, as it may be unfamiliar to some readers.
• **Highlight key findings**: Summarize what the research discovered about students' commognitive processes.
• Remove redundancy and improve flow for publication-level clarity.

Commented [H.2]: • Focus more tightly on the gap this study addresses—*how commognition helps understand students' thinking in PBL.*
• **Reduce general educational statements**: Shorten the generic educational goals and emphasize the theoretical framework and specific problem.
• Avoid repetition and unclear transitions; revise awkward or literal translations from Bahasa Indonesia.

Student worksheets are instructional tools that facilitate student learning and practice. This fosters an engaging collaboration between students and teachers. (Marsa et al., 2016). Student worksheets can support students in learning in a structured manner, thereby facilitating educators in creating enriched classroom experiences(Mulyani & F, 2020). Based on the aforementioned definitions, problem-based student worksheets can be considered as a model of student worksheets designed to enhance the effectiveness of learning. The development of problem-based learning student worksheets serves as a learning companion for students and is expected to improve the overall quality of education. Students' problem-solving processes can be analyzed through a commognitive approach (Presmeg, 2016).

According to Sfard, thinking is a process of communicating with oneself, either verbally or symbolically (Zayyadi & Pratiwi, 2022). Kata "*commognitive*" berasal dari kata "komunikasi" dan "kognitif", yang menunjukkan bahwa berpikir (atau berkomunikasi dengan diri sendiri) dan berkomunikasi dengan orang lain adalah hal yang sama (Sfard, 2016) . Zayyadi, et al. (2019) identifies four fundamental elements that constitute commognitive processes. First, the use of mathematical terminology through verbal expressions to represent concepts such as numbers, algebra, and equations. Second, the incorporation of visual mediators, including graphs, diagrams, and images, to facilitate mathematical reasoning. Third, the application of narratives to explain mathematical facts, encompassing axioms, definitions, formulas, and theorems. Lastly, the implementation of routines to describe the sequential steps undertaken in problem-solving processes (Setyowati et al., 2022).

Supardi et al., (2021) asserts that by analyzing commognitive processes, one can identify the challenges students face in problem-solving not only by examining the final outcomes but also by observing how words, visual mediators, narratives, and routines are utilized. In alignment with this perspective. D. Kim et al (2017) also asserts that the commognitive discourse analysis method has the potential to provide greater clarity in understanding students' mathematical thinking and the processes through which mathematics learning takes place in schools.

Commognitive analysis is conducted by researchers to examine how students communicate their knowledge in relation to Student Worksheets in Grade XI. This study contributes to education by advancing theoretical frameworks, particularly in enhancing the understanding of how cognitive processes and learning can be analyzed through problem-based learning Student Worksheets and the commognitive approach.

METODE

Commented [H.3]: • **Clarify research paradigm**: Is this a case study within exploratory qualitative design?
• **Detail instruments**: Clarify how worksheets were validated. Who assessed them for capturing commognitive components?
• **Justify sampling**: Briefly explain why selecting only one student per ability level is methodologically acceptable.
• **Ethics**: Include a sentence confirming ethical approval and informed consent.

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. The mathematical problem sheets and interview guidelines were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. The data collection techniques employed in this study aim to systematically obtain data to address the research questions. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

RESULTS AND DISCUSSION

Commented [H.4]: • **Streamline descriptions**: The analysis is overly long and sometimes repetitive, particularly in word use and visual mediator sections.
• **Use tables/figures strategically**: Consider summarizing observations across subjects in a comparison table for clarity.
• **Avoid redundant captions**: Replace repeated "Based on Figure X..." with varied phrasing.
• **Add interpretation**: Currently, much of the analysis is descriptive—add brief interpretive commentary within each subsection (e.g., "This indicates conceptual flexibility...").

Hasil Penelitian

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of

19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). To categorize the measurement results into three distinct levels, the following guidelines can be utilized as a reference:

Table 1. Formulas for Categorizing Student Ability Test Instruments

Ability Category	Formula
High	$X > M + 1SD$
Moderate	$M + 1SD < X \leq M - 1SD$
Low	$X \leq M - 1SD$

Based on the application of these formulas, the range of ability scores for each student was obtained, as shown in the following table.

Table 2. Results of Student Ability Test Instrument Categorization

Ability Category	Formula
High	$X > 63.8$
Moderate	$35{,}06 \leq X < 63{,}82$
Low	$X \leq 35{,}06$

High-Ability Subject

Based on the student worksheet completed by the subject NSU, a commognitive analysis will be conducted on the responses in the answer sheet and the transcript of the interview with the subject. This analysis will examine the four commognitive components: word use, visual mediators, routines, and narratives.

1. Word Use

Word use, or key terminology, refers to terms used in discourse, which can include mathematical terminology as well as non-mathematical terms related to everyday life. The word use identified in the high-ability subject (NSU) can be observed in Figure 1.

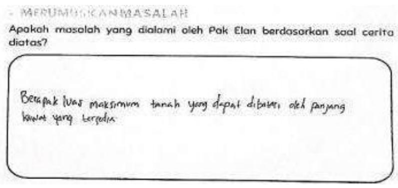
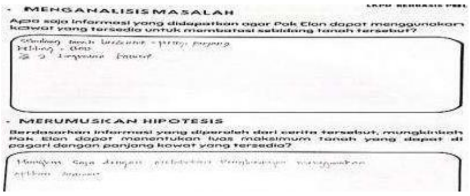


Figure 1. NSU Subject's Response Results

Based on Figure 1, it is evident that in the initial stage of problem formulation within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms area, maximum, and length. Furthermore, the word use utilized by the high-ability NSU subject in Figure 2 pertains to the stages of problem analysis and problem formulation.

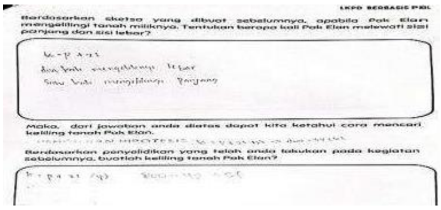


Gambar 2. Hasil Jawaban Subjek NSU

Based on Figure 2, it is evident that in the problem analysis stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms rectangle and perimeter. Additionally, in the hypothesis formulation stage, the NSU subject utilizes the word use application and derivative. The word use applied by the NSU subject in the data collection stage can be observed in Figure 3.

Figure 3. NSU Subject's Response Results

Based on Figure 3, it is evident that in the data collection stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms formula and length. In the hypothesis testing stage, the word use utilized by the NSU subject can be observed in Figure 4 below.



LKPD BERBASIS PBL

Berdasarkan sketsa yang dibuat sebelumnya, apabila Pak Elan mengelilingi tanah miliknya. Tentukan berapa kali Pak Elan melewati sisi panjang dan sisi lebar?

Maka, dari jawaban anda diatas dapat kita ketahui cara mencari keliling tanah Pak Elan.

Berdasarkan penyelidikan yang telah anda lakukan pada kegiatan sebelumnya, buatlah keliling tanah Pak Elan?

Gambar 4. Hasil Jawaban Subjek NSU

Based on Figure 4, it is evident that in the data collection stage within the problem-based learning (PBL) Student Worksheet, the word use employed by the NSU subject includes the terms two, one, width, and length.

The terminology used by the high-ability NSU subject in the problem-based learning (PBL) Student Worksheet can be categorized as both mathematical and non-mathematical, including terms related to everyday life. The mathematical terms identified in the NSU subject's responses include area, maximum, length, width, rectangle, one, two, derivative, mathematics, and perimeter. Meanwhile, the term that is non-mathematical or related to daily life, possessing multiple meanings, is kali.

2. *Visual Mediator*

The visual mediator in the responses provided by the NSU subject in the problem-based learning (PBL) Student Worksheet appears at various stages of problem-solving. In the hypothesis formulation stage, the iconic visual mediator used by the NSU subject can be observed in Figure 3, where the Student Worksheet contains depictions of land and a rectangular fence. To further interpret the meaning of these iconic visual mediators, an interview was conducted with the subject. Based on the interview results presented in Table 4, it was revealed that the NSU subject described the image as "a rectangular shape representing the land area."

Additionally, the symbolic visual mediator employed by the NSU subject is also evident in Figure 3, where, in the hypothesis formulation stage, the subject includes elements of the drawn image, such as **p** and **l**. In the data collection stage, the NSU subject writes the symbols **K, P,** and **l**. Similarly, in Figure 4, still within the data collection stage, the symbolic visual mediator consists of **K, P,** and **l**. During the hypothesis testing stage, as shown in Figure 4, the subject again employs the symbols **K, P,** and **l**. The visual mediators used by the NSU subject can be further examined in Figure 5 below.

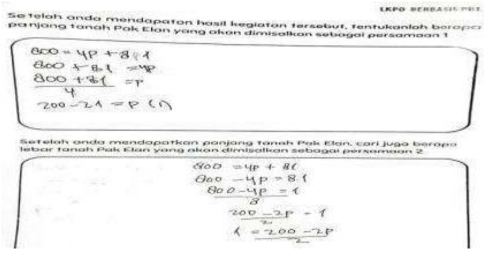


Figure 5. NSU Subject's Response Results

Based on Figure 5, it is evident that in the hypothesis testing stage, the symbolic visual mediators used by the NSU subject include K, P, and l. These symbols can also be observed in Figures 6 and 7.

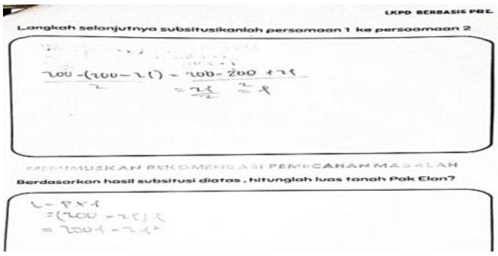


Figure 6. NSU Subject's Response Results

Based on Figure 6, the information obtained includes the use of the symbols **p** and **l**. The visual mediator used by the NSU subject can be further observed in Figure 7 below.

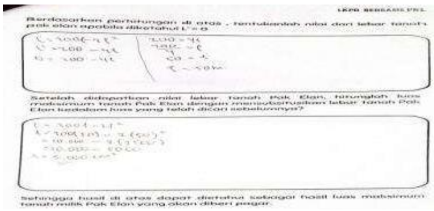


Figure 7. NSU Subject's Response Results

Based on Figure 7, the symbolic visual mediators identified include L, L', l, and m. According to the interview with the NSU subject, the subject explained the symbols used, stating, "So the symbols present here, the first ones are K, then P, and L." However, the concrete visual mediator was not observed in the NSU subject's responses in the Student Worksheet or the interview results. When asked about the absence of concrete visual mediators, the subject stated, "No, because there were no tools available." This indicates that the NSU subject did not employ or utilize concrete visual mediators. Based on these findings, it can be concluded that visual mediators appeared throughout various stages, including hypothesis formulation, data collection, hypothesis testing, and developing problem-solving recommendations.

3. *Routine*

Based on the responses written by Subject NSU in the Student Worksheets, from Image 1 to Image 7, the subject completed the PBL-based Student Worksheets by following the designated steps: formulating the problem, analyzing the problem, formulating a hypothesis, collecting data, testing the hypothesis, and formulating a recommendation for problem-solving. In Image 3, the subject wrote the formula for the perimeter of a rectangle as $P = p + 2l$, followed by the use of the area formula $A = p \times l$, and applied the derivative concept using $L' = 0$. As a result, the maximum area of Mr. Elan's land, enclosed with four levels of wire, was calculated to be 5,000 m².

Based on the interview results, it was found that Subject NSU explained, "I solved it by following each of the steps provided, and then created the formulas." The formulas used by Subject NSU included: "first the perimeter formula, then the area of the rectangle, and finally the formula for maximum area.

4. *Narrative*

To assess the mathematical understanding of Subject NSU, a high-ability student, the *narrative* component serves as a key indicator. Based on the written responses provided in the Student Worksheets, it was evident that Subject NSU accurately completed each of the required steps. In the first step, the subject successfully formulated the problem, followed by a correct analysis of the problem through identification of the relevant information provided in the worksheet. To gain deeper insight, both written responses and interview data were examined. The interview revealed that Subject NSU was able to recall and explain the given information, stating: "The information obtained from the problem includes, first, a rectangular piece of land, and second, a length of 800 meters, and umm… there will be a fence made with more than two levels." This understanding was also reflected in Subject NSU's response in Image 1.

In Image 3, Subject NSU applied the formula for the perimeter of a rectangle and, based on the scenario, chose a fence with four levels, thus multiplying the perimeter by four. In accordance with the steps outlined in the Student Worksheets, Subject NSU accurately and systematically calculated the length of the land owned by Mr. Elan. The subject also demonstrated precision in the subsequent step involving the calculation of the land's width.

Medium-Ability Subject

Based on the Student Worksheet completed by Subject PM, an analysis will be conducted to examine the commognitive components evident in the subject's responses, namely word use, visual mediators, routines, and narratives.

1. *Word Use*

Word use, or key terms, refers to expressions used in discourse, which may include both mathematical terminology and non-mathematical terms related to everyday life. The following *word use* identified in the medium-ability subject (PM) can be seen in Image 8.

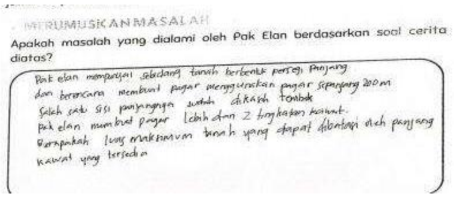


Figure 8. Results of the PM Subject Responses

The word use in the PM subject found in the responses of the Student Worksheets based on PBL can be seen in Figure 8. The information reveals that in the step of formulating the problem, the terms written by the PM subject are square, length, one, side, and area. In Figure 8, the question in the first step is about the problem statement experienced by Pak Elan; however, the subject also answered with information that appears in the question of the Student Worksheets. This indicates that the PM subject has not fully understood the intent of the first step, which is to formulate the problem.

Figure 9. Results of the PM Subject Responses

Based on Figure 9, in the step of analyzing the problem, the word use employed by the PM subject is the term "rectangle." In Figure 9, the step of analyzing the problem includes an instruction to mention all the information obtained. However, the NN subject only provided the information "K = 800 m and the land is in the shape of a rectangle" and did not mention any other information, as the remaining details were already included in the problem formulation section.

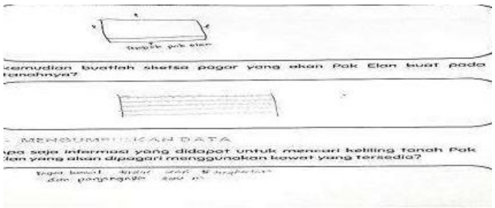
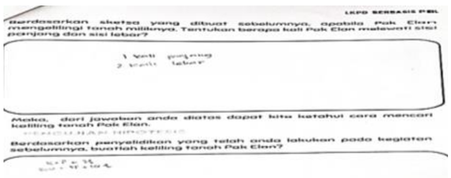


Figure 10. Results of the PM Subject Responses

Based on Figure 10, in the step of collecting data, it is evident that the word use employed by the NN subject is the term "length.



Gambar 11. Hasil Jawaban Subjek PM

Based on Figure 11, the word use found in it includes "times," "length," and "width." The terms used by the PM subject, a student with moderate ability, in the completed Student Worksheets can be categorized as either mathematical terms or terms not directly related to mathematics but connected to daily life. The terms that can be used mathematically by the PM subject include area, maximum, length, width, rectangle, times, and perimeter. On the other hand, the term "times" has dual meanings: mathematically, it refers to multiplication or calculation, while in other contexts, it can mean a river or stream (bengawan).

2. *Visual Mediator*

The visual mediator or visual mediator in the results of the Student Worksheets based on PBL for the PM subject with moderate ability, in the iconic visual mediator, can be seen in Figure 10. In the step of formulating the hypothesis, the PM subject created a drawing/sketch of Pak Elan's land and then designed a fence with five levels of wire. To understand the purpose of the image created by the PM subject, it can be found in the interview results, which state: "There is the image of Pak Elan's land that has been surrounded by a wall, and the second is a fence with five levels of wire. Meanwhile, the symbolic visual mediator found in the PM subject's response in Figure 10 is where the PM subject wrote the elements of the image, namely "p" and "l." Based on Figure 11, in the step of testing the hypothesis, the PM subject wrote the symbols "K," "P," and "l."

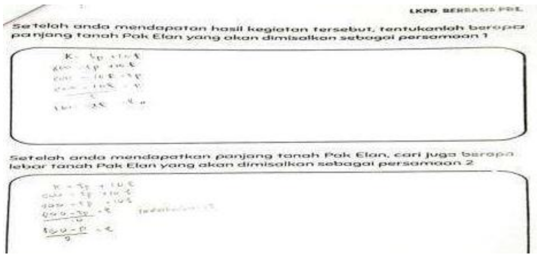


Figure 12. Results of the PM Subject Responses

Based on Figure 12, the information written by the PM subject includes the symbols "K," "p," and "l." In Figure 13, the PM subject wrote the symbols "l" and "L." The visual mediator of the PM subject can be seen in Figure 13 below.

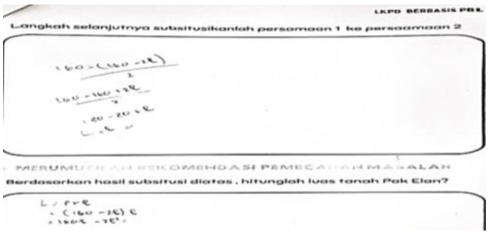


Figure 13. Results of the PM Subject Responses

Based on Figure 13, the visual mediator used by the PM subject in the stage of formulating problem-solving recommendations includes the symbols "L," "l," and "p," which are symbolic visual mediators. The visual mediator of the PM subject can be seen in Figure 14 below.

Figure 14. Results of the PM Subject Responses

Based on Figure 14, the PM subject wrote the symbol "L" as area (d), and "L'" was also found, along with the symbol for the unit "m," which represents meters. The concrete visual mediator of the PM subject is not present in the answers on the Student Worksheets based on PBL or in the interview results. The reason for this absence, according to the interview, is that the PM subject stated, "No, because I just wanted to finish quickly to catch up with time as I needed to complete other tasks." This indicates that the PM subject did not include or use any concrete visual media.

3. ***Routine***

The routine carried out by the PM subject in the results of the Student Worksheets based on PBL, as seen in Figures 8 to 14, is the same as that of the NSU subject. The PM subject completed the Student Worksheets by following each step outlined in the worksheets. The PM subject completed the Student Worksheets based on PBL according to the steps, starting with formulating the problem, analyzing the problem, formulating the hypothesis, collecting data, testing the hypothesis, and formulating problem-solving recommendations. In Figure 9, the subject wrote the formula for the perimeter of a rectangle, which is P + 2l. The subject then used the formula for the area of a rectangle, p x l, and applied the derivative formula with L' = 0. As a result, the maximum area of Pak Elan's land with five levels of wire was obtained as 3.200 m^2

4. ***Narrative***

The mathematical understanding of the PM subject, a student with moderate ability, can be seen from the narratives. The NN subject completed the Student Worksheets. Based on the completed PBL-based Student

Worksheets, in the problem formulation section, the NN subject provided a longer answer, but it was less accurate. In the step of analyzing the problem, the information written in the Student Worksheets shows that the PM subject only wrote "K = 800 m" and "the land is in the shape of a rectangle." The information written by the NN subject is less detailed, but the information that should have been explained in the analysis section was explained in the problem formulation section instead. Next, the PM subject followed the steps by creating a drawing. In the drawing made by the PM subject, more than two pieces of information were written, and five levels were taken into account. In the data collection section, the PM subject answered correctly. Following this, for each step in the Student Worksheets, the PM subject wrote the answers in accordance with the instructions provided.

Low Ability Subject

Based on the PBL-based Student Worksheet completed by the AR subject, a commognitive analysis will be conducted on the responses provided by the subject, focusing on the elements of word use, visual mediator, routine, and narrative.

1. Word Use

Based on Figure 15, the information in the step of formulating the problem reveals that the word use observed in the Student Worksheets responses from the AR subject is "rectangle.

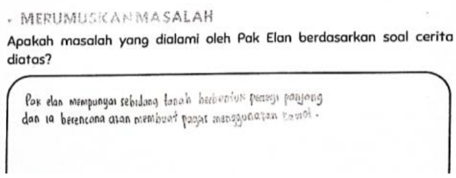


Figure 15. Results of the PM Subject Responses

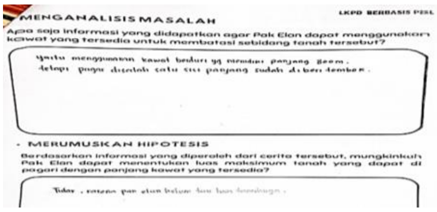


Figure 16. Results of the PM Subject Responses

Based on Figure 16, the information indicates that in the problem analysis step, the observed word use includes "length," "side," and "one.

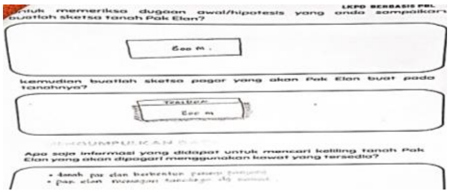


Figure 17. Results of the PM Subject Responses

Figure 18. Results of the PM Subject Responses

Based on Figure 18, the information reveals that the AR subject used the terms "times," "perimeter," "side," "length," and "width." The terms employed by the AR subject, a low-ability student, in the completed Student Worksheets can be categorized as both mathematical terms and non-mathematical terms, including those related to everyday life. The mathematical terms used by the AR subject are the same as those found in the PM subject's responses, including "area," "length," "width," "rectangle," "side," "times," and "perimeter." However, the term "times" (kali) has a dual meaning: in a mathematical context, it refers to multiplication or calculation, whereas in everyday language, it can mean "river" or "stream" (bengawan).

2. *Visual Mediator*

The visual mediator in the responses of the PBL-based Student Worksheets for the AR subject, a low-ability student, can be observed in Figure 16. In the hypothesis formulation step, the AR subject created a drawing or sketch of Pak Elan's land and then depicted a fence with four levels of wire. The purpose of the drawing made by the AR subject can be understood through the interview response: “Yes, I drew Pak Elan’s land and the fence. Meanwhile, the symbolic visual mediator found in the AR subject's response is observed in Figure 18, in the hypothesis testing step, where the AR subject wrote the symbols K, P, and l.

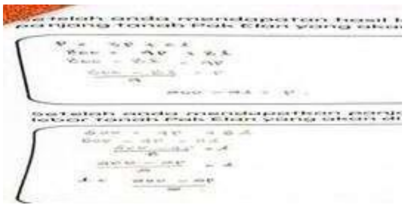


Figure 19. Results of the AR Subject Responses

Based on Figure 19, in the hypothesis testing step, the symbols written by the AR subject are **p** and **l**. Meanwhile, in Figure 20, the symbols written are the same as those in Figure 19, namely **p** and **l**, but an additional symbol **L** is also present, representing area.

Figure 20. Results of the AR Subject Responses

Based on Figure 20, in the stage of formulating and recommending a problem-solving approach, the visual mediator used by the AR subject includes the symbols l, P, and L.

Figure 21. Results of the AR Subject Responses

Based on Figure 21, the information indicates that the AR subject wrote the symbols L' and l. The concrete visual mediator of the AR subject is not present in the responses on the PBL-based Student Worksheets.

3. ***Routine***

The routine followed by the AR subject in completing the PBL-based Student Worksheets, as observed in Figures 14 to 21, demonstrates that the subject adhered to each step outlined in the worksheets. The AR subject completed the Student Worksheets by following the structured steps, starting with formulating the problem, analyzing the problem, formulating a hypothesis, collecting data, testing the hypothesis, and formulating problem-solving recommendations. Based on Figure 15, in the problem formulation step, the AR subject did not provide a correct response to the question. In Figure 16, during the problem analysis step, only minimal information was provided by the AR subject. In Figure 17, the AR subject created a drawing in the hypothesis formulation step. In Figure 18, the AR subject applied the formula for the perimeter of a rectangle, P + 2l. Subsequently, the subject used the formula for the area of a rectangle, p × l. However, the AR subject was unable to correctly apply the derivative formula with L' = 0, leading to an incorrect final result.

4. ***Narrative***

The mathematical understanding of the AR subject, a student with moderate ability, can be analyzed through narratives. The NN subject completed the Student Worksheets. Based on the PBL-based Student Worksheets completed by the AR subject, it is evident that in the problem formulation section, the response provided by the AR subject was incorrect. In the problem analysis step, the information recorded in the Student Worksheets shows that the AR subject only wrote "the length of the wire is 800 m and one of its sides has already been walled." The information provided by the AR subject was minimal and lacked detail. Next, the AR subject followed the structured steps by creating a drawing. In the drawing, the AR subject included more than two pieces of information, incorporating four levels. In the data collection section, the AR subject provided accurate responses. Throughout the subsequent steps in the Student Worksheets, the AR subject followed the given instructions correctly. However, despite following the procedural steps, the AR subject made an error in deriving the final result.

Diskusi

Based on the research findings on the commognitive analysis of students in solving problem-based learning (PBL) Student Worksheets, which involve the commognitive components of word use, visual mediator, narrative, and routine, data were obtained from students of Grade XI at SMA Muhammadiyah Bangkinang. The findings provide insights into how students employ these components in problem-solving.

NSU Subject – High-Ability Student

Based on the research findings, the NSU subject demonstrated all four commognitive components (word use, visual mediator, routine, and narrative). In terms of word use, the NSU subject employed both mathematical and non-mathematical terms, as observed in the Student Worksheets and interview responses. The NSU subject used seven mathematical terms and one non-mathematical term. The problem-solving stages followed by the NSU subject reflect appropriate word use, with accurate and well-structured responses provided

Commented [H.5]: • **Structure clearly by themes**: Currently reads like an extended result summary. Instead, organize by key insights (e.g., “Differences in Word Use Across Ability Levels,” “Visual Mediators and Abstract Thinking”).
• **Critical comparison**: Engage more deeply with contrasting findings from literature (e.g., how your findings agree or disagree with Setyowati et al., 2022).
• **Theoretical contribution**: Highlight how your study extends commognitive theory into PBL contexts or Indonesian classrooms.

at each step. The application of word use by the NSU subject was consistently well-executed, ensuring completeness in each stage of the problem-solving process (Zayyadi et al., 2023). The NSU subject repeatedly demonstrated word use during problem formulation, problem analysis, hypothesis formulation, data collection, and hypothesis testing.

The NSU subject demonstrated both iconic and symbolic visual mediators by drawing the shape of Pak Elan's land and fence. Within the land diagram, the NSU subject incorporated elements in the form of symbols such as p and l, along with additional symbols like K, L, L', and M. However, the NSU subject did not employ concrete visual media, stating in the interview that using additional media was unnecessary for completing the Student Worksheets. According to Jean Piaget's theory, research subjects aged 12 years and above fall within the formal operational stage, where individuals develop the ability to think abstractly and systematically (Marinda, 2020). At the formal operational stage, a key characteristic is that children no longer require concrete objects or events to aid their thinking; instead, they develop the ability to think abstractly. In terms of routine, the NSU subject applied formulas for the perimeter of a rectangle, the area of a rectangle, and the maximum area using derivative applications. The NSU subject demonstrated routine by systematically following each step in the Student Worksheets according to the instructions provided by the subject teacher. This aligns with the perspective of Mudaly & Mpofu (2019). Ritualized routines are the result of ritualized learning, where procedures are committed to memory, recalled, and mechanically applied when a specific situation requires their implementation. The NSU subject also demonstrated explorative routine, as they were able to modify the perimeter formula into $p + 2l$ and demonstrated an understanding of its conceptual application. Mudaly & Mpofu, (2019) states that further exploration and activities should be utilized to develop these routines into a deeper understanding, allowing students to recognize various practical applications of the concept. Furthermore, routine applicability was observed in the NSU subject, as they used several symbols such as p, l, and k and also incorporated diagrams in their responses to complete the PBL-based Student Worksheets. However, routine flexibility was also evident, as the NSU subject applied multiple formulas to solve the PBL-based Student Worksheets, including the perimeter formula for a rectangle, the area formula for a rectangle, and the derivative application formula.

The narrative of the NSU subject indicates that they followed each stage outlined in the PBL-based Student Worksheets. The NSU subject also demonstrated a clear understanding of the application of each step. This is evident in the completed Student Worksheets, where the responses provided by the NSU subject are well-structured and clearly presented (Rossydha et al., 2021). The student used "I" as the object level to describe the routine they followed in completing the tasks. This indicates a personal engagement with the problem-solving process, demonstrating an awareness of the steps taken and the reasoning behind them.

It can be concluded that the NSU subject, in the responses provided in the problem-based learning (PBL) Student Worksheets on the topic of derivative applications, demonstrated all four commognitive components. Within the Student Worksheets, the NSU subject systematically and accurately completed each section, following the steps correctly and appropriately. This finding aligns with research conducted by Setyowati et al., (2022) highlighted the emergence of all four commognitive components in students with a visual learning style. Similarly, the NSU subject has demonstrated the ability to effectively communicate their thought processes, both internally and in interactions with others (Rossydha et al., 2021). Through the commognitive framework, the NSU subject has demonstrated the ability to communicate their thoughts in a structured and logical manner.(Setyowati et al., 2022).

Subject PM – Students with Moderate Ability

Based on the research findings, it was found that Subject PM demonstrated all four commognitive components—word use, visual mediators, routines, and narratives. In terms of word use, Subject PM employed both mathematical and non-mathematical terms, as evident in the Student Worksheets and interview responses. The word use observed in Subject PM appeared predominantly during the problem comprehension stage (Zayyadi et al., 2023). However, Subject PM demonstrated inaccuracies in completing the steps outlined in the Student Worksheets, particularly in the sections concerning problem formulation and problem analysis.

Visual mediators function as tools for presenting problems through visual representations, including images or diagrams (Setyowati et al., 2022). Subject PM utilized both iconic and symbolic visual mediators by drawing the shape of the land and the fence. Within the land illustration, the subject included the symbols p and q to represent specific elements of the problem. Similar to Subject NSU, Subject PM did not use any concrete visual media, stating that the use of additional materials was unnecessary for completing the Student WorksheetsAccording to Jean Piaget's theory as cited in Marinda, (2020) research subjects aged 12 years and above are generally in the formal operational stage. This stage is characterized by the individual's ability to think without relying on concrete objects or events. During this developmental period, children are capable of abstract reasoning and can mentally manipulate ideas and hypothetical situations.

In the aspect of routine, Subject PM applied formulas for the perimeter and area of a rectangle, as well as the formula for maximum area found in the application of derivatives. This indicates that Subject PM demonstrated routine flexibility, as evidenced by the use of more than one method or formula to solve the problem. Additionally, Subject PM employed ritual routines by following step-by-step procedures outlined in the Student Worksheets and adhering to the instructional methods previously taught by the subject-matter teacher (Mudaly & Mpofu, 2019). Exploratory routines and extended activities should be employed to further develop these routines into deeper conceptual understanding, enabling learners to recognize various practical applications of the concept (Mudaly & Mpofu, 2019). In this context, the exploratory routine demonstrated by Subject PM was similar to that of Subject NSU, as both were able to modify the formula for the perimeter of a rectangle. Subject PM was also able to determine calculations based on a fencing model involving five levels of wire. Furthermore, routine applicability was evident in Subject PM's work, as the subject utilized several symbols such as p, l, and k, and included a drawing in the response sheet to complete the PBL-based Student Worksheet.

The narrative of Subject PM was similar to that of Subject NSU in that Subject PM did not fully follow each step outlined in the Student Worksheets. An examination of the completed Student Worksheets revealed that Subject PM's responses in the problem formulation and problem analysis sections were not entirely accurate. However, during the interview, Subject PM was able to articulate and demonstrate an understanding of the tasks completed. In the subsequent steps, Subject PM showed a clear understanding of how to apply each stage in solving the problem, including the correct use of procedures and formulas, as confirmed through interview data. As a result, the final version of the Student Worksheets completed by the subject appeared more systematic.

As a result, the final version of the Student Worksheets completed by the subject appeared more systematic. It can therefore be concluded that Subject PM, in the completed Student Worksheets, demonstrated all four components of commognitive theory, in line with the findings reported by Setyowati et al., (2022) identified all four components of commognitive theory within the context of visual learning styles. In the case of Subject PM, the Student Worksheets were completed fairly well; however, there was a lack of full understanding and accuracy in one particular step—namely, the formulation and analysis of the problem. Based on the interview results, Subject PM demonstrated the ability to analyze the problem effectively, suggesting that the subject's interpersonal communication was stronger than their intrapersonal communication.

Subject AR: Learners with Limited Academic Proficiency

Based on the research findings, Subject AR demonstrated all four components of commognitive theory: word use, visual mediators, routines, and narratives. In terms of word use, Subject AR employed both mathematical and non-mathematical terminology, which appeared in the Student Worksheets as well as in the interview responses. The use of mathematical language by Subject AR began at the problem formulation stage, during which the subject used seven mathematical terms and one non-mathematical term.

Subject AR employed both iconic and symbolic visual mediators by drawing the shape of the land and the surrounding fence. Similar to Subjects PM and NSU, Subject AR did not utilize any concrete visual media, stating that there was no need to use additional materials, as drawing directly in the Student Worksheets was considered sufficient.

Subject AR applied formulas for the perimeter and area of a rectangle, as well as the formula for maximum area within the context of derivative applications. This indicates that Subject AR demonstrated routine flexibility, as evidenced by the use of multiple methods or formulas to solve the problem. In addition, Subject AR exhibited ritual routines by following the procedural steps outlined in the Student Worksheets and those taught by the subject-matter teacher. Exploratory routines were also evident, as Subject AR was able to develop and modify several formulas, including the formula for the perimeter of a rectangle. Mudaly & Mpofu, (2019) stated that exploration and extended activities should be used to develop routines into deeper conceptual understanding, enabling learners to recognize various practical applications of the concept. In line with this, routine applicability was evident in Subject AR's work, as the subject used several symbols such as p, l, and k, and included a drawing in the response sheet to complete the PBL-based Student Worksheet.

Subject AR followed the steps outlined in the Student Worksheets; however, the subject did not fully understand the use of each step and the application of relevant procedures. This was evident from the completed Student Worksheets, particularly in the problem formulation step, where the response was inaccurate, and in the problem analysis section, where only limited information was provided.

Nevertheless, based on the interview results, Subject AR was able to articulate relevant information and demonstrate an ability to analyze the problem. In the final step of the Student Worksheets, Subject AR's answer was incorrect due to a misunderstanding of the concept of derivative applications. However, the subject showed an adequate understanding of the formulas used, indicating a partial grasp of their conceptual application.

It can therefore be concluded that Subject AR demonstrated all four components of commognitive theory in the completed Student Worksheets, consistent with the findings reported in the study conducted by Setyowati et al., (2022) identified all four components of commognitive theory in relation to visual and kinesthetic learning styles. Based on the completed Student Worksheets, Subject AR showed some similarities with Subject PM, particularly in demonstrating a relatively good level of completion. Furthermore, interview data indicated that Subject AR was capable of analyzing the problem, suggesting that both intrapersonal and interpersonal communication skills were adequately developed. In the final step, however, Subject AR provided an incorrect final answer in the Student Worksheets. Nevertheless, the subject demonstrated a reasonable understanding of the procedural steps and the application of formulas required to complete the task.

CONCLUSION

Commented [H.6]: • **Avoid repetition**: Don't reiterate full results. Synthesize into two or three key conclusions.
• **Implications**: Add pedagogical implications for teachers or curriculum developers.
• **Future research**: Suggest areas for further inquiry.

Based on the presentation of research findings and discussion, it can be concluded that the high-ability subject demonstrated all four components of commognitive theory within the context of the problem-based learning Student Worksheets. The subject employed *word use* that included both mathematical and non-mathematical terms. The *visual mediators* used were both iconic and symbolic in nature. In terms of *routines*, the subject applied formulas related to the area and perimeter of rectangles, as well as the formula for maximum area in the context of derivatives. The subject also engaged in both *ritual* and *exploratory* routines. The *narrative* component was reflected in the subject's demonstrated understanding of how to complete the Student Worksheets and apply the appropriate procedures and formulas.

The medium-ability subject demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. However, in the step of problem formulation, the subject included some information that overlapped with the data collection phase, resulting in limited written input during the data-gathering section. Consequently, the *word use* component appeared only to a minimal extent. The subject used both mathematical and non-mathematical terms. The *visual mediators* employed by the medium-ability subject were similar to those used by the high-ability subject, consisting of both iconic and symbolic representations. Regarding *routines*, the subject applied formulas for the area and perimeter of rectangles, as well as for determining the maximum area using derivatives. Similar to the high-ability subject, the medium-ability subject also demonstrated both *ritual* and *exploratory* routines. The *narrative* component was reflected in the subject’s understanding of how to complete the problem-based learning Student Worksheets and apply the relevant procedures and formulas.

The low-ability subject also demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. The subject used both mathematical and non-mathematical terminology as part of the *word use* component. The *visual mediators* employed were iconic and symbolic, similar to those used by the high-ability subject. In terms of *routines*, the subject engaged in both *ritual* and *exploratory* routines. However, the subject provided an incorrect answer in the final step due to a misunderstanding of the concept of derivative applications. The *narrative* component was evident in the subject’s understanding of several procedural steps within the Student Worksheets and a partial grasp of the formulas and procedures used.

It can therefore be concluded that all three subjects—those with high, medium, and low ability—demonstrated the four components of commognitive theory in completing the problem-based learning Student Worksheets. The use of problem-based learning worksheets had a positive impact on students' communication processes and their thinking in relation to problem-solving.

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Commented [H.7]: • **Check for consistency**: Some references are incomplete (e.g., Kim et al., 2017 lacks title).
• **Update citations**: Ensure recent and peer-reviewed sources are prioritized over local conference proceedings unless essential.
• **Apply APA 7th Edition or journal's citation format**: Ensure uniform punctuation, italics, DOI formatting.

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

Commognitive is a term that combines communication and cognition that emphasizing the principle that interpersonal communication and individual thinking are two sides of the phenomenon. This study analyzes students' commognitive processes in solving problem-based learning worksheets. The research employed an exploratory descriptive approach and was conducted at SMA Muhammadiyah. The research subjects were students from class XI IPA 2 at SMA Muhammadiyah Bangkinang. The sampling technique used was purposive sampling, selecting three students as research subjects: one with high ability, one with moderate ability, and one with low ability. This sampling strategy is appropriate in qualitative research, particularly within case study designs, as it allows for in-depth exploration of variations in student thinking across ability levels. Data were collected through written tests, interviews, and documentation. The researcher developed the problem-based worksheets and interview guides to capture the four key components of commognitive theory—word use, visual mediators, narratives, and routines—which refer to how students communicate and engage in mathematical thinking through discourse. These instruments were validated by mathematics education experts to ensure they were capable of eliciting the intended commognitive components. The data analysis techniques included data collection, data reduction, data presentation, and conclusion drawing. The findings revealed variations in the commognitive processes of students based on their ability levels. High-ability students demonstrated more coherent narratives and consistent use of mathematical routines, while students with moderate and low ability exhibited more fragmented discourse and inconsistencies in the use of visual mediators and word use. Before conducting the study, ethical approval was obtained, and all participants provided informed consent by ethical research standards.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios. However, understanding how students engage with these materials requires a deeper analysis of their thinking processes.

This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication. Zayyadi, et al. (2019) identify four core elements in the commognitive framework: (1) word use—students' use of mathematical terminology to express concepts; (2) visual mediators—the use of diagrams, symbols, or images in reasoning; (3) narratives—how students construct mathematical statements, such as definitions and theorems; and (4) routines—the patterns or steps students take in solving problems (Setyowati et al., 2022).

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. This selection represents a purposeful sampling strategy commonly used in case study designs within exploratory qualitative research, allowing for in-depth exploration of diverse student experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. The mathematical problem sheets and interview guidelines were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. These instruments were validated through expert judgment involving mathematics education specialists, who assessed the extent to which the tasks could elicit the intended commognitive components.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). To categorize the measurement results into three distinct levels, the following guidelines can be utilized as a reference:

Table 1. Formulas for Categorizing Student Ability Test Instruments

Ability Category	Formula
High	$X > M + 1SD$
Moderate	$M + 1SD < X \leq M - 1SD$
Low	$X \leq M - 1SD$

Based on the application of these formulas, the range of ability scores for each student was obtained, as shown in the following table.

Table 2. Results of Student Ability Test Instrument Categorization

Ability Category	Formula
High	$X > 63.8$
Moderate	$35{,}06 \leq X < 63{,}82$
Low	$X \leq 35{,}06$

High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time). This shows conceptual flexibility and fluency in math discourse.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K, P, l, L') throughout all problem-solving steps. Absence of concrete mediators was acknowledged due to unavailability of tools. This indicates visual literacy and symbol manipulation skills.

Routine

NSU followed PBL steps systematically, applying formulas and derivatives (e.g., $L' = 0$) to find a maximum area of 5,000 m². This demonstrates a structured problem-solving strategy.

Narrative

NSU accurately recalled and applied information, reasoning logically through each worksheet step and interview. The narrative confirms a thorough understanding and metacognitive awareness.

Moderate-Ability Subject (PM)

Word Use

PM used correct terms like area, side, and length, though sometimes misaligned them with instructions (e.g., giving known information instead of formulating a problem). This suggests partial understanding.

Visual Mediators

Both iconic and symbolic mediators were used (e.g., sketches, symbols p, l, K, L'). No concrete media was used, with time constraints cited. Indicates an ability to represent math visually, though possibly under pressure.

Routine

PM completed all steps, derived formulas, and calculated a 3,200 m^2 area using L' = 0. This shows procedural understanding but with minor conceptual slips.

Narrative

PM followed instructions but often provided information in the wrong sections. Still, the process of drawing and formula use shows emerging understanding.

Low-Ability Subject (AR)

Word Use

AR used relevant terms (e.g., rectangle, length, perimeter), with evidence of dual-meaning terms (e.g., kali). Some responses lacked precision, reflecting limited mathematical vocabulary usage.

Visual Mediators

AR used sketches and symbolic mediators (e.g., P, l, L, L') in most steps but did not use concrete visualizations. Demonstrates basic visual representation ability, though with limited elaboration.

Routine

AR followed the worksheet structure but struggled with correct formula application—especially with L' = 0—leading to incorrect conclusions.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 3. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors
Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

Discussion

Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023).

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023).

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p, l, and k. However, none used concrete visual media, aligning with Piaget's formal operational stage in adolescents (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Despite structural accuracy in drawings, conceptual weaknesses appeared in early worksheet stages.

Limited-Ability Student (Subject AR)

Visual representation was consistent with other subjects but lacked precise conceptual integration, contributing to misunderstandings in final solutions.

Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and derivative-based maximum area.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative routines. For instance, they modified perimeter formulas and logically sequenced steps, reflecting procedural fluency (Mudaly & Mpofu, (2019).

Moderate-Ability Student (Subject PM)

Subject PM showed routine flexibility and was able to adapt formulas (e.g., converting perimeter formulas). However, execution was inconsistent, especially in early-stage problem comprehension.

Limited-Ability Student (Subject AR)

Subject AR demonstrated formula adaptation and exploratory thinking. However, misunderstanding of derivative applications in the final step indicated partial conceptual grasp.

Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step. They used the pronoun "I" to reflect on their process, showcasing metacognitive awareness (Rossydha et al., 2021).

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. Although early steps were unclear, interviews revealed retrospective understanding and corrected application in later steps.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative understanding. The final answer was incorrect, though interviews suggested a moderate ability to reflect and analyze.

Critical Comparison with Literature

The findings support Setyowati et al., (2022), who found all four commognitive components evident in students with visual learning styles. However, this study reveals nuances in how ability level influences the depth and accuracy of these components. For example, while Setyowati et al. observed general component use, this study highlights discrepancies in conceptual application, especially among moderate- and low-performing students.

Furthermore, the ritualized routines described by (Mudaly & Mpofu, (2019). were evident across all participants, but only NSU demonstrated a shift toward exploratory and flexible routine use—a contrast with Setyowati et al.'s broader generalization.

Theoretical Contribution

This study extends Sfard's commognitive theory into Indonesian PBL contexts, providing empirical evidence on how commognitive elements manifest differently across ability levels. It also suggests that commognition is influenced not only by individual cognition but also by educational structure, cultural context, and teacher instruction in Indonesian classrooms. Notably, the study underscores how PBL can surface varying depths of discourse among learners, supporting the theory's relevance in student-centered learning environments.

4. CONCLUSION

Based on the presentation of research findings and discussion, it can be concluded that the high-ability subject demonstrated all four components of commognitive theory within the context of the problem-based learning Student Worksheets. The subject employed word use that included both mathematical and non-mathematical terms. The visual mediators used were both iconic and symbolic in nature. In terms of routines, the subject applied formulas related to the area and perimeter of rectangles, as well as the formula for maximum area in the context of derivatives. The subject also engaged in both ritual and exploratory routines. The narrative component was reflected in the subject's demonstrated understanding of how to complete the Student Worksheets and apply the appropriate procedures and formulas.

The medium-ability subject demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. However, in the step of problem formulation, the subject included some information that overlapped with the data collection phase, resulting in limited written input during the data-gathering section. Consequently, the word use component appeared only to a minimal extent. The subject used both mathematical and non-mathematical terms. The visual mediators employed by the medium-ability subject were similar to those used by the high-ability subject, consisting of both iconic and symbolic representations. Regarding routines, the subject applied formulas for the area and perimeter of rectangles, as well as for determining the maximum area using derivatives. Similar to the high-ability subject, the medium-ability subject also demonstrated both ritual and exploratory routines. The narrative component was reflected in the subject's understanding of how to complete the problem-based learning Student Worksheets and apply the relevant procedures and formulas.

The low-ability subject also demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. The subject used both mathematical and non-mathematical terminology as part of the word use component. The visual mediators employed were iconic and symbolic, similar to those used by the high-ability subject. In terms of routines, the subject engaged in both ritual and exploratory routines. However, the subject provided an incorrect answer in the final step due to a misunderstanding of the concept of derivative applications. The narrative component was evident in the subject's understanding of several procedural steps within the Student Worksheets and a partial grasp of the formulas and procedures used.

In summary, all three subjects—high, medium, and low ability—exhibited the four components of commognitive theory through their engagement with the problem-based learning Student Worksheets. This indicates the effectiveness of problem-based learning in fostering mathematical communication and cognitive development across diverse student abilities.

Pedagogical Implications

These findings suggest that problem-based learning materials should be designed to emphasize all four commognitive components—word use, visual mediators, routines, and narratives—to support mathematical understanding at varying levels of student ability. Teachers and curriculum developers should consider incorporating more structured scaffolding for lower-ability students, particularly in applying conceptual knowledge such as derivatives, to prevent misconceptions.

Future Research Directions

Further research could explore how specific design elements of problem-based learning worksheets influence each commognitive component. Additionally, longitudinal studies could examine how repeated exposure to such worksheets impacts students' mathematical discourse over time. Investigating teacher-student interactions during worksheet implementation could also offer deeper insights into how commognitive development is facilitated in classroom practice.

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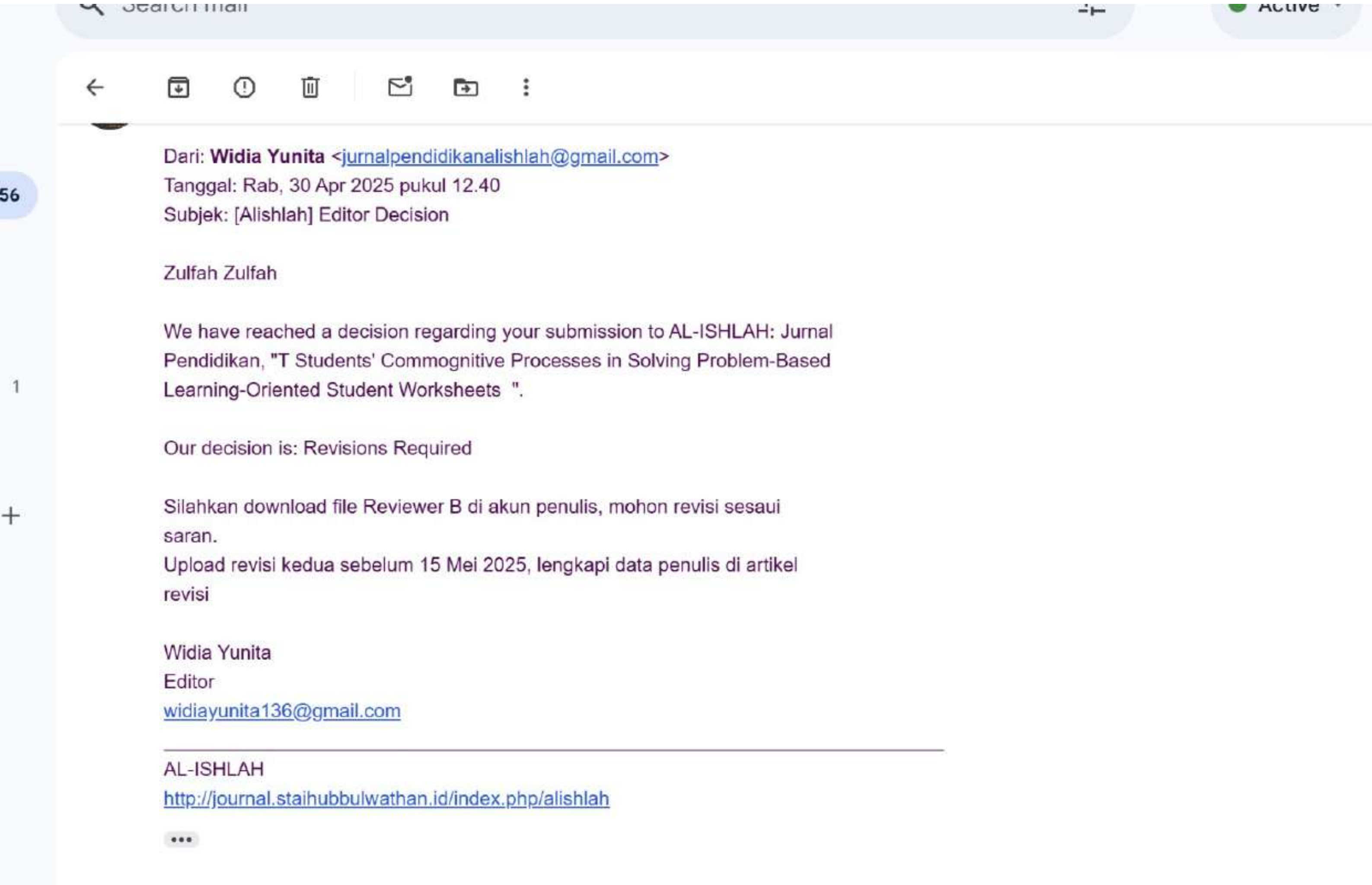

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

Commognitive is a term that combines communication and cognition that emphasizing the principle that interpersonal communication and individual thinking are two sides of the phenomenon. This study analyzes students' commognitive processes in solving problem-based learning worksheets. The research employed an exploratory descriptive approach and was conducted at SMA Muhammadiyah. The research subjects were students from class XI IPA 2 at SMA Muhammadiyah Bangkinang. The sampling technique used was purposive sampling, selecting three students as research subjects: one with high ability, one with moderate ability, and one with low ability. This sampling strategy is appropriate in qualitative research, particularly within case study designs, as it allows for in-depth exploration of variations in student thinking across ability levels. Data were collected through written tests, interviews, and documentation. The researcher developed the problem-based worksheets and interview guides to capture the four key components of commognitive theory—word use, visual mediators, narratives, and routines—which refer to how students communicate and engage in mathematical thinking through discourse. These instruments were validated by mathematics education experts to ensure they were capable of eliciting the intended commognitive components. The data analysis techniques included data collection, data reduction, data presentation, and conclusion drawing. The findings revealed variations in the commognitive processes of students based on their ability levels. High-ability students demonstrated more coherent narratives and consistent use of mathematical routines, while students with moderate and low ability exhibited more fragmented discourse and inconsistencies in the use of visual mediators and word use. Before conducting the study, ethical approval was obtained, and all participants provided informed consent by ethical research standards.

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Commognitive theory; mathematical discourse; student worksheets; problem-based learning; qualitative analysis

Abstract: 150-250 words
- Needs rephrasing for clarity, grammar, and cohesion.
- Include clearer statements on:
a. Purpose
b. Methodology
c. Key findings
d. Contributions to practice/theory

Introduction:
- Clearly establish the research problem: *What gap in commognitive research or PBL practice is being addressed?*
- Move the definition of "commognition" earlier.
- Briefly define each component (word use, mediators, etc.) and why these are important in PBL.

Method:
- **Clarify sample justification**: Explain *why* one student per ability level is sufficient.
- **Add validation detail**: Who were the validators? What rubric or protocol was used?
- **Instrument clarity**: Provide sample worksheet questions or describe the type of mathematical problems posed.

Findings and Discussion:
- Avoid repetition across subjects; synthesize findings.
- **Table 1 and Table 2** are redundant; consider merging or simplifying.
- **Strengthen Discussion**:
  - Link findings more explicitly to Piaget's theory and other learning theories.
  - Highlight what is novel or different from prior commognitive studies.
  - Include a clearer reflection on *what the findings mean for classroom teaching* (beyond just reporting what happened).

Conclusion:
- Clarify theoretical and practical implications separately.
- Ensure final paragraph clearly answers: *What should educators or researchers do next based on these findings?*

1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios. However, understanding how students engage with these materials requires a deeper analysis of their thinking processes.

This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication. Zayyadi, et al. (2019) identify four core elements in the commognitive framework: (1) word use—students' use of mathematical terminology to express concepts; (2) visual mediators—the use of diagrams, symbols, or images in reasoning; (3) narratives—how students construct mathematical statements, such as definitions and theorems; and (4) routines—the patterns or steps students take in solving problems (Setyowati et al., 2022).

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. This selection represents a purposeful sampling strategy commonly used in case study designs within exploratory qualitative research, allowing for in-depth exploration of diverse student experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. The mathematical problem sheets and interview guidelines were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. These instruments were validated through expert judgment involving mathematics education specialists, who assessed the extent to which the tasks could elicit the intended commognitive components.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). To categorize the measurement results into three distinct levels, the following guidelines can be utilized as a reference:

Table 1. Formulas for Categorizing Student Ability Test Instruments

Ability Category	Formula
High	$X > M + 1SD$
Moderate	$M + 1SD < X \leq M - 1SD$
Low	$X \leq M - 1SD$

Based on the application of these formulas, the range of ability scores for each student was obtained, as shown in the following table.

Table 2. Results of Student Ability Test Instrument Categorization

Ability Category	Formula
High	$X > 63.8$
Moderate	$35{,}06 \leq X < 63{,}82$
Low	$X \leq 35{,}06$

High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time). This shows conceptual flexibility and fluency in math discourse.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K, P, l, L') throughout all problem-solving steps. Absence of concrete mediators was acknowledged due to unavailability of tools. This indicates visual literacy and symbol manipulation skills.

Routine

NSU followed PBL steps systematically, applying formulas and derivatives (e.g., L' = 0) to find a maximum area of 5,000 m². This demonstrates a structured problem-solving strategy.

Narrative

NSU accurately recalled and applied information, reasoning logically through each worksheet step and interview. The narrative confirms a thorough understanding and metacognitive awareness.

Moderate-Ability Subject (PM)

Word Use

PM used correct terms like area, side, and length, though sometimes misaligned them with instructions (e.g., giving known information instead of formulating a problem). This suggests partial understanding.

Visual Mediators

Both iconic and symbolic mediators were used (e.g., sketches, symbols p, l, K, L'). No concrete media was used, with time constraints cited. Indicates an ability to represent math visually, though possibly under pressure.

Routine

PM completed all steps, derived formulas, and calculated a 3,200 m^2 area using L' = 0. This shows procedural understanding but with minor conceptual slips.

Narrative

PM followed instructions but often provided information in the wrong sections. Still, the process of drawing and formula use shows emerging understanding.

Low-Ability Subject (AR)

Word Use

AR used relevant terms (e.g., rectangle, length, perimeter), with evidence of dual-meaning terms (e.g., kali). Some responses lacked precision, reflecting limited mathematical vocabulary usage.

Visual Mediators

AR used sketches and symbolic mediators (e.g., P, l, L, L') in most steps but did not use concrete visualizations. Demonstrates basic visual representation ability, though with limited elaboration.

Routine

AR followed the worksheet structure but struggled with correct formula application—especially with L' = 0—leading to incorrect conclusions.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 3. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors
Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

Discussion

Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023).

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023).

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p, l, and k. However, none used concrete visual media, aligning with Piaget's formal operational stage in adolescents (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Despite structural accuracy in drawings, conceptual weaknesses appeared in early worksheet stages.

Limited-Ability Student (Subject AR)

Visual representation was consistent with other subjects but lacked precise conceptual integration, contributing to misunderstandings in final solutions.

Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and derivative-based maximum area.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative routines. For instance, they modified perimeter formulas and logically sequenced steps, reflecting procedural fluency (Mudaly & Mpofu, (2019).

Moderate-Ability Student (Subject PM)

Subject PM showed routine flexibility and was able to adapt formulas (e.g., converting perimeter formulas). However, execution was inconsistent, especially in early-stage problem comprehension.

Limited-Ability Student (Subject AR)

Subject AR demonstrated formula adaptation and exploratory thinking. However, misunderstanding of derivative applications in the final step indicated partial conceptual grasp.

Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step. They used the pronoun "I" to reflect on their process, showcasing metacognitive awareness (Rossydha et al., 2021).

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. Although early steps were unclear, interviews revealed retrospective understanding and corrected application in later steps.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative understanding. The final answer was incorrect, though interviews suggested a moderate ability to reflect and analyze.

Critical Comparison with Literature

The findings support Setyowati et al., (2022), who found all four commognitive components evident in students with visual learning styles. However, this study reveals nuances in how ability level influences the depth and accuracy of these components. For example, while Setyowati et al. observed general component use, this study highlights discrepancies in conceptual application, especially among moderate- and low-performing students.

Furthermore, the ritualized routines described by (Mudaly & Mpofu, (2019). were evident across all participants, but only NSU demonstrated a shift toward exploratory and flexible routine use—a contrast with Setyowati et al.'s broader generalization.

Theoretical Contribution

This study extends Sfard's commognitive theory into Indonesian PBL contexts, providing empirical evidence on how commognitive elements manifest differently across ability levels. It also suggests that commognition is influenced not only by individual cognition but also by educational structure, cultural context, and teacher instruction in Indonesian classrooms. Notably, the study underscores how PBL can surface varying depths of discourse among learners, supporting the theory's relevance in student-centered learning environments.

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In summary, all three subjects—high, medium, and low ability—exhibited the four components of commognitive theory through their engagement with the problem-based learning Student Worksheets. This indicates the effectiveness of problem-based learning in fostering mathematical communication and cognitive development across diverse student abilities.

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Further research could explore how specific design elements of problem-based learning worksheets influence each commognitive component. Additionally, longitudinal studies could examine how repeated exposure to such worksheets impacts students' mathematical discourse over time. Investigating teacher-student interactions during worksheet implementation could also offer deeper insights into how commognitive development is facilitated in classroom practice.

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

This study explores students' commognitive processes—integrating communication and cognition—during the completion of problem-based learning worksheets in mathematics. Grounded in commognitive theory, which views interpersonal communication and individual thinking as interconnected, the research aims to analyze how students of varying ability levels engage in mathematical discourse. Employing an exploratory descriptive approach, the study was conducted at SMA Muhammadiyah Bangkinang with participants drawn from class XI IPA 2. Using purposive sampling, three students were selected to represent high, moderate, and low mathematical ability. Data were collected through written tests, interviews, and documentation, using researcher-developed worksheets and interview guides aligned with the four commognitive components: word use, visual mediators, narratives, and routines. These instruments were validated by experts in mathematics education. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing. The findings reveal that high-ability students exhibited more coherent narratives and consistent use of mathematical routines. In contrast, students with moderate and low ability demonstrated fragmented discourse and less effective use of visual mediators and terminology. This study contributes to the understanding of how students' thinking processes vary across ability levels and offers insights into the design of learning tools that support the development of mathematical discourse. Ethical approval was obtained, and all participants provided informed consent in accordance with research ethics standards.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources

is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios.

Understanding how students engage with these materials requires a deeper analysis of their thinking processes. This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication.

Commognitive theory identifies four core elements of student discourse by Zayyadi, et al. (2019): (1) word use—how students employ mathematical terminology to express concepts; (2) visual mediators—such as diagrams, symbols, or images used in reasoning; (3) narratives—students' construction of mathematical statements like definitions or theorems; and (4) routines—the recognizable patterns or steps students apply when solving problems (Setyowati et al., 2022). These components are particularly important in a PBL context, as they provide a structured lens through which to analyze students' engagement with complex, real-world tasks.

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The selection of one student per ability level is justified by the case study design's focus on in-depth, context-rich exploration of individual experiences, which is a core strength of qualitative research. A sample size of three allows for a comparative analysis across diverse levels of ability while maintaining depth and thoroughness in exploring each student's experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. These instruments were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. To ensure the validity of these instruments, they were subjected to expert judgment. The validators were two mathematics education specialists and one expert in cognitive development. The rubric used for validation involved assessing the tasks' ability to provoke the four key commognitive components and whether they aligned with the research objectives. Additionally, the worksheet's clarity, mathematical relevance, and capacity to engage students were evaluated.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this approach is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

Sample questions on the worksheet aimed to assess students' mathematical reasoning in problem-solving contexts. For example, one of the problems involved a real-world scenario requiring the application of algebraic reasoning to solve a practical problem, such as determining the cost of materials given a budget constraint. Other problems involved visual aids like diagrams to assess spatial reasoning and interpretations of mathematical relationships. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). The ability levels were determined using the following formulas:

Table 1. Categorization of Student Abilities Based on Scores.

Ability Category	Formula	Score Range
High	X > M + 1SD	X > 63.8
Moderate	M + 1SD < X ≤ M - 1SD	35.06 ≤ X < 63.82
Low	X ≤ M - 1SD	X ≤ 35.06

High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time). This shows conceptual flexibility and fluency in math discourse.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K, P, l, L') throughout all problem-solving steps. Absence of concrete mediators was acknowledged due to unavailability of tools. This indicates visual literacy and symbol manipulation skills.

Routine

NSU followed PBL steps systematically, applying formulas and derivatives (e.g., L' = 0) to find a maximum area of 5,000 m². This demonstrates a structured problem-solving strategy.

Narrative

NSU accurately recalled and applied information, reasoning logically through each worksheet step and interview. The narrative confirms a thorough understanding and metacognitive awareness.

Moderate-Ability Subject (PM)

Word Use

PM used correct terms like area, side, and length, though sometimes misaligned them with instructions (e.g., giving known information instead of formulating a problem). This suggests partial understanding.

Visual Mediators

Both iconic and symbolic mediators were used (e.g., sketches, symbols p, l, K, L'). No concrete media was used, with time constraints cited. Indicates an ability to represent math visually, though possibly under pressure.

Routine

PM completed all steps, derived formulas, and calculated a 3,200 m² area using L' = 0. This shows procedural understanding but with minor conceptual slips.

Narrative

PM followed instructions but often provided information in the wrong sections. Still, the process of drawing and formula use shows emerging understanding.

Low-Ability Subject (AR)

Word Use

AR used relevant terms (e.g., rectangle, length, perimeter), with evidence of dual-meaning terms (e.g., kali). Some responses lacked precision, reflecting limited mathematical vocabulary usage.

Visual Mediators

AR used sketches and symbolic mediators (e.g., P, l, L, L') in most steps but did not use concrete visualizations. Demonstrates basic visual representation ability, though with limited elaboration.

Routine

AR followed the worksheet structure but struggled with correct formula application—especially with L' = 0—leading to incorrect conclusions.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 2. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors

Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

Discussion

Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023). This precision in mathematical language aligns with the formal operational stage of Piaget's theory, where abstract and systematic thinking becomes prominent.

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023). This suggests that while PM may be transitioning into formal operational thinking, there remains reliance on concrete strategies.

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers. This supports Vygotsky's notion that language use without internalized meaning may not reflect actual cognitive development, emphasizing the need for scaffolded instruction.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p, l, and k. However, none used concrete visual media, which aligns with Piaget's formal operational stage, where individuals are expected to think abstractly without relying on tangible objects (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding. This aligns with Sfard's view of visual mediators as integral tools for internal communication of mathematical concepts.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Although structurally accurate, their diagrams did not fully support the reasoning process, suggesting limitations in transforming visual tools into deeper understanding.

Limited-Ability Student (Subject AR)

Subject AR's visual representation was similar to the others but lacked conceptual integration, contributing to misinterpretations in the final solution. The student may have entered the formal operational stage but struggled to use visual mediators as vehicles for logical abstraction.

Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and maximum area via derivatives. This suggests that procedural knowledge was present across ability levels, but the depth of understanding varied.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative thinking. They modified perimeter formulas and logically sequenced steps, reflecting high procedural fluency and supporting (Mudaly & Mpofu, (2019) observation on adaptable routine use. Unlike the typical ritual routines seen in many students, NSU shifted toward personalized strategies—demonstrating a novel contribution to commognitive literature.

Moderate-Ability Student (Subject PM)

PM displayed some routine flexibility (e.g., converting formulas), but execution was inconsistent. The conceptual gaps suggest a need for guided discovery learning to foster more autonomous thinking.

Limited-Ability Student (Subject AR)

Subject AR showed formula adaptation but struggled in derivative application—indicating an incomplete grasp of higher-order routines. This highlights the need for more explicit modeling and metacognitive prompting in classroom teaching.

Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step using first-person pronouns, which signals metacognitive awareness (Rossydha et al., 2021). This reflects a high level of internal dialogue, consistent with Sfard's concept of thinking as communication with the self.

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. While initial steps lacked clarity, retrospective explanations in interviews showed emerging cognitive control and reflection—suggesting growth through social interaction, in line with Vygotsky's sociocultural theory.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative formation. The final answer was incorrect, yet interview responses revealed emerging reflection skills. This underscores the potential of narrative scaffolding in enhancing self-explanation and conceptual clarity.

Critical Comparison with Literature

The findings affirm Setyowati et al., (2022),, who reported that all commognitive components are present in students with visual learning styles. However, this study contributes novel insights by showing that ability level influences not just the presence, but the *depth* and *accuracy* of these components. Unlike prior studies which generalized commognitive use, this research demonstrates that high-ability students engage more flexibly and metacognitively with mathematical discourse.

Furthermore, the ritualized routines discussed by (Mudaly & Mpofu, (2019) were evident across all participants, but only NSU transitioned toward exploratory and adaptive routine use—a deviation from previous findings. This supports the idea that routine development is mediated by both cognitive ability and instructional design.

Theoretical Contribution

This study extends Sfard's commognitive theory by applying it within the context of Indonesian problem-based learning (PBL) classrooms, offering empirical evidence on how commognitive elements manifest differently across student ability levels. Importantly, it highlights that commognition is not solely an individual process, but is shaped by instructional models, cultural norms, and teacher discourse.

By situating this analysis within a PBL framework, the study shows how structured yet open-ended learning can surface diverse depths of mathematical discourse. These findings underscore the importance of designing learning environments that allow for differentiated student expression, and encourage teachers to attend not only to student answers, but also to the how and why behind their reasoning. This has practical implications for lesson planning, formative assessment, and the integration of reflective discourse in mathematics classrooms.

4. CONCLUSION

Based on the presentation of research findings and discussion, it can be concluded that the high-ability subject demonstrated all four components of commognitive theory within the context of the problem-based learning Student Worksheets. The subject employed word use that included both mathematical and non-mathematical terms. The visual mediators used were both iconic and symbolic in nature. In terms of routines, the subject applied formulas related to the area and perimeter of rectangles, as well as the formula for maximum area in the context of derivatives. The subject also engaged in both ritual and exploratory routines. The narrative component was reflected in the subject's demonstrated understanding of how to complete the Student Worksheets and apply the appropriate procedures and formulas.

The medium-ability subject demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. However, in the step of problem formulation, the subject included some information that overlapped with the data collection phase, resulting in limited written input during the data-gathering section. Consequently, the word use component appeared only to a minimal extent. The subject used both mathematical and non-mathematical terms. The visual mediators employed by the medium-ability subject were similar to those used by the high-ability subject, consisting of both iconic and symbolic representations. Regarding routines, the subject applied formulas for the area and perimeter of rectangles, as well as for determining the maximum area using derivatives. Similar to the high-ability subject, the medium-ability subject also demonstrated both ritual and exploratory routines. The narrative component was reflected in the subject’s understanding of how to complete the problem-based learning Student Worksheets and apply the relevant procedures and formulas.

The low-ability subject also demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. The subject used both mathematical and non-mathematical terminology as part of the word use component. The visual mediators employed were iconic and symbolic, similar to those used by the high-ability subject. In terms of routines, the subject engaged in both ritual and exploratory routines. However, the subject provided an incorrect answer in the final step due to a misunderstanding of the concept of derivative applications. The narrative component was evident in the subject’s understanding of several procedural steps within the Student Worksheets and a partial grasp of the formulas and procedures used.

In summary, all three subjects—high, medium, and low ability—exhibited the four components of commognitive theory through their engagement with the problem-based learning Student Worksheets. This indicates the effectiveness of problem-based learning in fostering mathematical communication and cognitive development across diverse student abilities.

Theoretical Implications

These findings contribute to the extension of Sfard’s commognitive theory by demonstrating that students of varying abilities can exhibit the theory’s four components—word use, visual mediators, routines, and narratives—within a structured PBL environment. The study provides empirical support for the commognitive approach in exploring how mathematical thinking is constructed and communicated during problem-solving, especially in secondary-level mathematics education.

Practical Implications

From a pedagogical perspective, the study suggests that problem-based learning materials should be designed intentionally to activate all four commognitive components. Teachers and curriculum developers should incorporate scaffolding strategies—especially for lower-ability students—to support more accurate application of mathematical concepts, particularly in abstract topics such as derivatives. Emphasizing clarity in student narratives and symbolic representations may also improve conceptual integration and reduce misconceptions.

Recommendations for Educators and Researchers

Educators are encouraged to integrate commognitive elements explicitly into classroom instruction and assessment, using PBL worksheets as tools to reveal and develop student thinking. Researchers should investigate how variations in worksheet design or teacher facilitation influence each commognitive component. Future studies could also explore the long-term development of student discourse through repeated PBL exposure, and examine teacher-student interactions as key drivers of commognitive growth in classroom settings.

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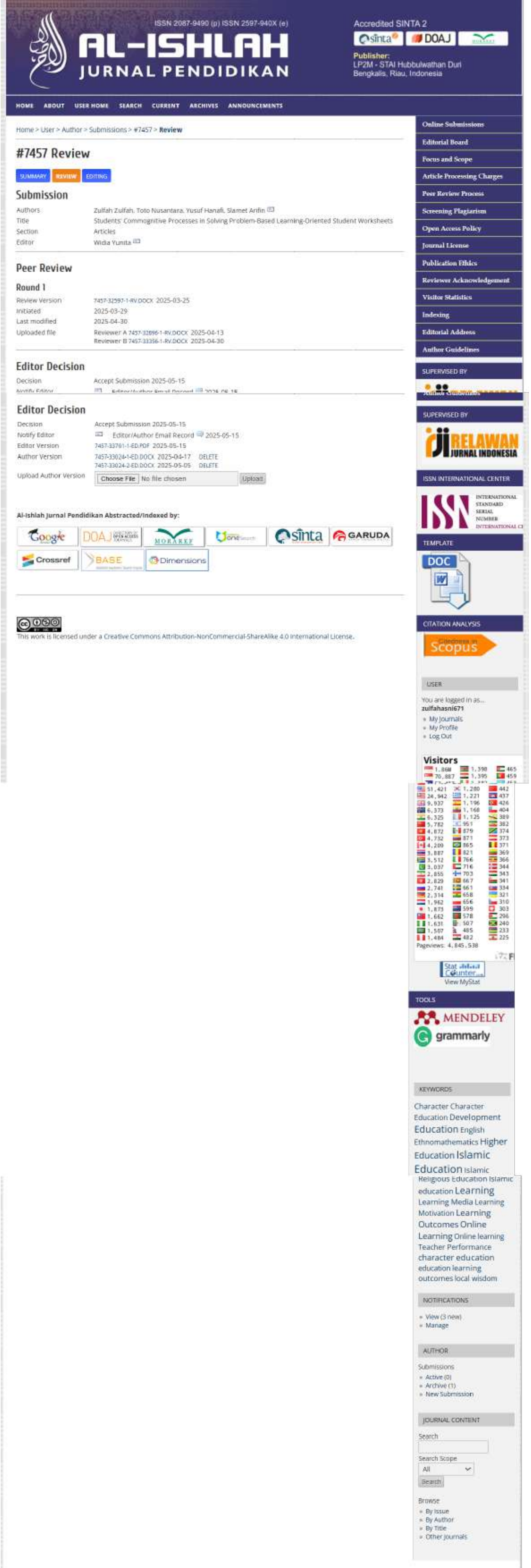

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#7457 Review

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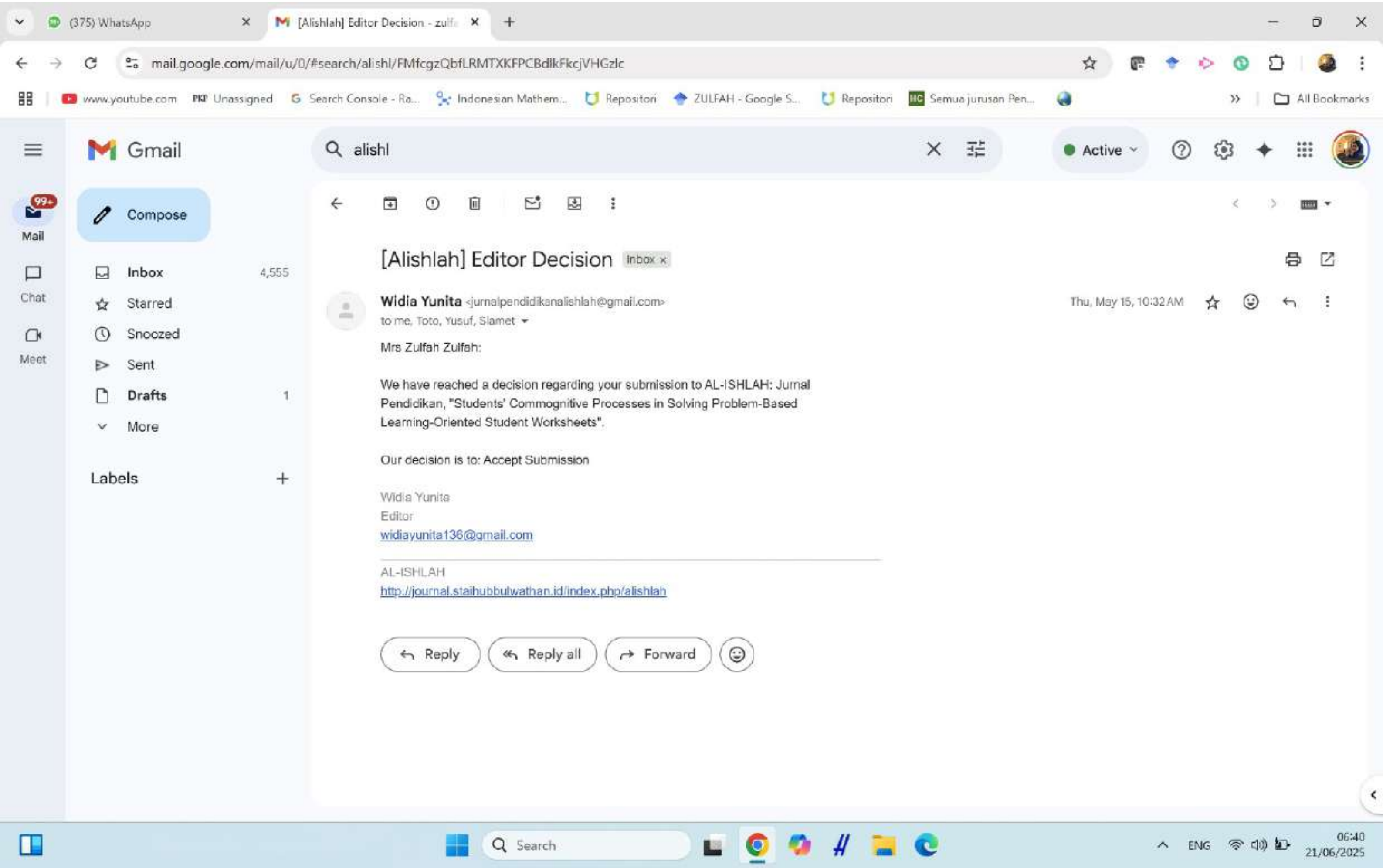

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Dear Zulfah, Toto Nusantara, Yusuf Hanafi, Slamet Arifin

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Students' Commognitive Processes in Solving Problem-Based Learning-Oriented Student Worksheets

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ABSTRACT

This study investigates students' *commognitive* processes—the integration of communication and cognition—during problem-based learning (PBL) in mathematics. It is grounded in *commognitive theory*, which posits that thinking and communication are interdependent. An exploratory descriptive approach was applied at SMA Muhammadiyah Bangkinang, focusing on students in class XI IPA 2. Using purposive sampling, three students representing high, moderate, and low mathematical abilities were selected. Data were collected through written tests, interviews, and documentation. Researcher-developed worksheets and interview protocols, aligned with the four commognitive components—word use, visual mediators, narratives, and routines—were validated by mathematics education experts. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing. Findings indicate that high-ability students demonstrated more coherent narratives and consistent application of mathematical routines. Conversely, students with moderate and low abilities exhibited fragmented discourse and less effective use of visual mediators and terminology. The study highlights the role of commognitive factors in shaping students' mathematical discourse across ability levels. It suggests that instructional tools designed with commognitive principles can better support the development of mathematical thinking. These insights contribute to understanding the cognitive-communicative dynamics in mathematics education and inform the design of targeted learning interventions. Ethical approval was obtained, and informed consent was secured from all participants.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources

is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios.

Understanding how students engage with these materials requires a deeper analysis of their thinking processes. This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication.

Commognitive theory identifies four core elements of student discourse by Zayyadi, et al. (2019): (1) word use—how students employ mathematical terminology to express concepts; (2) visual mediators—such as diagrams, symbols, or images used in reasoning; (3) narratives—students' construction of mathematical statements like definitions or theorems; and (4) routines—the recognizable patterns or steps students apply when solving problems (Setyowati et al., 2022). These components are particularly important in a PBL context, as they provide a structured lens through which to analyze students' engagement with complex, real-world tasks.

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The selection of one student per ability level is justified by the case study design's focus on in-depth, context-rich exploration of individual experiences, which is a core strength of qualitative research. A sample size of three allows for a comparative analysis across diverse levels of ability while maintaining depth and thoroughness in exploring each student's experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. These instruments were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. To ensure the validity of these instruments, they were subjected to expert judgment. The validators were two mathematics education specialists and one expert in cognitive development. The rubric used for validation involved assessing the tasks' ability to provoke the four key commognitive components and whether they aligned with the research objectives. Additionally, the worksheet's clarity, mathematical relevance, and capacity to engage students were evaluated.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this approach is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

Sample questions on the worksheet aimed to assess students' mathematical reasoning in problem-solving contexts. For example, one of the problems involved a real-world scenario requiring the application of algebraic reasoning to solve a practical problem, such as determining the cost of materials given a budget constraint. Other problems involved visual aids like diagrams to assess spatial reasoning and interpretations of mathematical relationships. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

3.1 Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). The ability levels were determined using the following formulas:

Table 1. Categorization of Student Abilities Based on Scores.

Ability Category	Formula	Score Range
High	$X > M + 1SD$	$X > 63.8$
Moderate	$M + 1SD < X \leq M - 1SD$	$35.06 \leq X < 63.82$
Low	$X \leq M - 1SD$	$X \leq 35.06$

3.1.1 High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time), demonstrating conceptual flexibility and fluency in mathematical discourse. This blending of technical and colloquial language suggests a strong ability to shift between abstract reasoning and practical application, which is essential for effective problem-solving. By incorporating both types of vocabulary, NSU not only shows command over formal mathematical concepts but also an awareness of how these ideas relate to real-world contexts. This ability to code-switch between different registers of language highlights a deeper understanding of mathematics beyond rote memorization.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K, P, l, L') throughout all problem-solving steps. The absence of concrete mediators was acknowledged, attributed to the unavailability of physical tools or manipulatives. This indicates not only visual literacy and symbol manipulation skills, but also adaptability in the face of material constraints. By effectively using sketches and abstract notations, NSU demonstrated an ability to visualize spatial relationships and translate them into formal representations. This strategic use of mediators reveals a capacity to shift between different semiotic systems, which is critical for advanced mathematical thinking and modeling.

Routine

NSU followed the Problem-Based Learning (PBL) steps systematically, applying appropriate formulas and calculus techniques, such as setting the derivative L′ =0L′ =0, to find a maximum area of 5,000 m^2 . This demonstrates a structured and goal-oriented problem-solving strategy. NSU's methodical approach reflects not only procedural fluency but also a solid conceptual understanding of optimization principles. By accurately translating the real-world context into a mathematical model and solving it analytically, NSU exhibited strong reasoning skills and the ability to navigate between abstract mathematics and practical application. This disciplined execution of PBL stages further underscores an ability to work independently and think critically within a structured framework.

Narrative

NSU accurately recalled and applied relevant information, reasoning logically through each worksheet step and verbalizing thought processes during the interview. The narrative confirms a thorough understanding of the mathematical content and reveals a high degree of metacognitive awareness. NSU consistently monitored progress, evaluated the appropriateness of strategies, and adjusted approaches when necessary—hallmarks of effective self-regulation. This reflective engagement suggests not only mastery of the material but also the ability to think critically about one's own thinking. Such metacognitive control is essential for deep learning and long-term academic success.

3.1.2 Moderate-Ability Subject (PM)

Word Use

PM used correct mathematical terms such as *area*, *side*, and *length*, indicating familiarity with foundational vocabulary. However, these terms were sometimes misaligned with the task instructions—for example, providing known information instead of independently formulating a problem. This suggests a partial understanding of the concepts and their functional use within problem-solving contexts. While PM demonstrates surface-level knowledge, the inconsistent application indicates difficulty in transferring this knowledge to new or unfamiliar tasks. The tendency to focus on recall over construction of meaning implies a need for further development in conceptual comprehension and task interpretation. Addressing this gap could support more independent and flexible problem-solving in the future.

Visual Mediators

Both iconic and symbolic mediators were used effectively—for example, sketches to represent spatial elements and symbols such as *p*, *l*, *K*, and *L*′ to articulate mathematical relationships. No concrete media (e.g., manipulatives or physical models) were utilized, with time constraints cited as a limiting factor. This indicates a capacity to represent mathematical ideas visually and abstractly, suggesting visual literacy and fluency in mathematical notation. The reliance on drawings and symbols in the absence of tangible tools also demonstrates flexibility and resourcefulness under pressure. However, the lack of concrete representation might limit opportunities for hands-on exploration, which can support deeper conceptual development, especially under time constraints. Overall, the mediator choices reflect a preference for abstract and visual strategies, possibly shaped by situational demands.

Routine

PM completed all problem-solving steps, derived appropriate formulas, and correctly calculated an area of 3,200 m^2 using the optimization technique L′=0L' = 0L′=0. This reflects a solid grasp of procedural steps and the mechanical application of calculus methods. However, minor conceptual slips were noted—such as imprecise interpretation of variables or partial understanding of the conditions under which maximum values occur. While PM followed the correct procedures, there appeared to be some reliance on rote methods rather than deep conceptual reasoning. This suggests that while procedural fluency is developing well, further support is needed to strengthen conceptual understanding and enhance the ability to explain *why* certain steps are taken, not just *how*.

Narrative

PM followed the task instructions but frequently placed information in incorrect sections, suggesting some confusion about the structure or expectations of the worksheet. Despite this, the use of appropriate drawings and formulas demonstrates an emerging understanding of the mathematical concepts involved. PM's attempts to represent the problem visually and symbolically indicate developing skills in mathematical modeling. These efforts reflect a growing awareness of the relationship between visual representation, algebraic formulation, and problem-solving. While structural organization needs improvement, the willingness to engage with multiple representations is a promising sign of conceptual growth and increasing mathematical confidence.

3.1.3 Low-Ability Subject (AR)

Word Use

AR used relevant mathematical terms such as *rectangle*, *length*, and *perimeter*, demonstrating familiarity with foundational geometry vocabulary. Notably, the use of dual-meaning terms like *kali* indicated attempts to bridge everyday language with mathematical ideas, revealing an intuitive but informal grasp of key concepts. However, several responses lacked precision, reflecting a limited and occasionally inconsistent use of mathematical vocabulary. This suggests that while AR possesses a basic understanding of geometric terms, there may be difficulty in selecting the most accurate or context-appropriate terminology. The blending of informal and formal language highlights the need for targeted vocabulary development to support clearer and more precise mathematical communication. Strengthening this aspect would enhance AR's ability to construct and convey mathematical reasoning more effectively.

Visual Mediators

AR used both sketches and symbolic mediators—such as P, l, L, and L'—in most steps of the problem-solving process, indicating an emerging ability to represent mathematical ideas visually and symbolically. However, there was no use of concrete visualizations (e.g., manipulatives, physical models, or real-life analogies), which may have limited deeper conceptual engagement. While the sketches and symbols suggest familiarity with abstract forms of representation, they lacked detailed elaboration or explanatory connections between visual elements and mathematical reasoning. This indicates a basic level of visual representation skill, but one that is still developing in terms of clarity, purpose, and integration with problem-solving strategies. Encouraging AR to connect visual tools more intentionally with conceptual understanding could enhance both communication and comprehension.

Routine

AR followed the worksheet structure consistently, indicating an understanding of procedural expectations and task organization. However, difficulties emerged with the correct application of formulas—particularly the use of the derivative condition L′=0L' = 0L′=0—which led to incorrect or incomplete conclusions. This suggests a gap in understanding the underlying principles of optimization, such as recognizing when and why a derivative is set to zero to find a maximum or minimum. Although AR attempted to engage with formal mathematical methods, the misapplication reveals a reliance on surface-level procedures rather than conceptual reasoning. Targeted support in interpreting and applying calculus-based strategies would help AR move beyond mechanical execution toward deeper problem-solving competence.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 2. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic are used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors
Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

3.2 Discussion

3.2.2 Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023). This precision in mathematical language aligns with the formal operational stage of Piaget's theory, where abstract and systematic thinking becomes prominent.

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023). This suggests that while PM may be transitioning into formal operational thinking, there remains reliance on concrete strategies.

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers. This supports Vygotsky's notion that language use without internalized meaning may not reflect actual cognitive development, emphasizing the need for scaffolded instruction.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p, l, and k. However, none used concrete visual media, which aligns with Piaget's formal operational stage, where individuals are expected to think abstractly without relying on tangible objects (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding. This aligns with Sfard's view of visual mediators as integral tools for the internal communication of mathematical concepts.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Although structurally accurate, their diagrams did not fully support the reasoning process, suggesting limitations in transforming visual tools into deeper understanding.

Limited-Ability Student (Subject AR)

Subject AR's visual representation was similar to the others but lacked conceptual integration, contributing to misinterpretations in the final solution. The student may have entered the formal operational stage but struggled to use visual mediators as vehicles for logical abstraction.

3.2.2 Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and maximum area via derivatives. This suggests that procedural knowledge was present across ability levels, but the depth of understanding varied.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative thinking. They modified perimeter formulas and logically sequenced steps, reflecting high procedural fluency and supporting (Mudaly & Mpofu, (2019) observation on adaptable routine use. Unlike the typical ritual routines seen in many students, NSU shifted toward personalized strategies—demonstrating a novel contribution to commognitive literature.

Moderate-Ability Student (Subject PM)

PM displayed some routine flexibility (e.g., converting formulas), but execution was inconsistent. The conceptual gaps suggest a need for guided discovery learning to foster more autonomous thinking.

Limited-Ability Student (Subject AR)

Subject AR showed formula adaptation but struggled in derivative application—indicating an incomplete grasp of higher-order routines. This highlights the need for more explicit modeling and metacognitive prompting in classroom teaching.

3.2.3 Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step using first-person pronouns, which signals metacognitive awareness (Rossydha et al., 2021). This reflects a high level of internal dialogue, consistent with Sfard's concept of thinking as communication with the self.

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. While initial steps lacked clarity, retrospective explanations in interviews showed emerging cognitive control and reflection—suggesting growth through social interaction, in line with Vygotsky's sociocultural theory.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative formation. The final answer was incorrect, yet interview responses revealed emerging reflection skills. This underscores the potential of narrative scaffolding in enhancing self-explanation and conceptual clarity.

3.2.4 Critical Comparison with Literature

The findings affirm Setyowati et al. (2022), who reported that all commognitive components are present in students with visual learning styles. However, this study contributes novel insights by showing that ability level influences not just the presence, but the *depth* and *accuracy* of these components. Unlike prior studies which generalized commognitive use, this research demonstrates that high-ability students engage more flexibly and metacognitively with mathematical discourse.

Furthermore, the ritualized routines discussed by (Mudaly & Mpofu, 2019) were evident across all participants, but only NSU transitioned toward exploratory and adaptive routine use—a deviation from previous findings. This supports the idea that routine development is mediated by both cognitive ability and instructional design.

3.2.5 Theoretical Contribution

This study extends Sfard's commognitive theory by applying it within the context of Indonesian problem-based learning (PBL) classrooms, offering empirical evidence on how commognitive elements manifest differently across student ability levels. Importantly, it highlights that commognition is not solely an individual process, but is shaped by instructional models, cultural norms, and teacher discourse.

By situating this analysis within a PBL framework, the study shows how structured yet open-ended learning can surface diverse depths of mathematical discourse. These findings underscore the importance of designing learning environments that allow for differentiated student expression, and encourage teachers to attend not only to student answers, but also to the how and why behind their reasoning. This has practical implications for lesson planning, formative assessment, and the integration of reflective discourse in mathematics classrooms.

4. CONCLUSION

This study found that all three students—high, medium, and low ability—showed evidence of the four components of commognitive theory (word use, visual mediators, routines, and narratives) while working on problem-based learning (PBL) Student Worksheets. The high-ability student used both mathematical and everyday language, applied correct formulas, used both symbolic and iconic visuals, and demonstrated strong understanding through both exploratory and ritual routines. The medium-ability student also showed all four components, but had difficulty separating the steps of problem formulation and data collection, which limited the clarity of their word use. The low-ability student demonstrated basic understanding of all four components but made errors applying derivatives, showing the need for further conceptual support. Overall, the research supports that PBL can help students of different abilities express mathematical thinking through communication and problem-solving.

However, the study had some limitations. It only involved three students, so the findings may not represent a wider group. Also, the research focused only on written worksheets and individual interviews, without examining how teacher support or peer discussions might influence students' commognitive development.

Future research should explore how changes in worksheet design or teacher guidance impact the development of each commognitive component. It would also be valuable to study how students' mathematical communication evolves over time with repeated exposure to PBL, and to observe classroom interactions to understand how teachers support student thinking. Educators are encouraged to use PBL worksheets to draw out students' mathematical reasoning and to provide extra support, especially in helping lower-ability students apply abstract concepts like derivatives more accurately.

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ABSTRACT

This study explores students' commognitive processes—integrating communication and cognition—during the completion of problem-based learning worksheets in mathematics. Grounded in commognitive theory, which views interpersonal communication and individual thinking as interconnected, the research aims to analyze how students of varying ability levels engage in mathematical discourse. Employing an exploratory descriptive approach, the study was conducted at SMA Muhammadiyah Bangkinang with participants drawn from class XI IPA 2. Using purposive sampling, three students were selected to represent high, moderate, and low mathematical ability. Data were collected through written tests, interviews, and documentation, using researcher-developed worksheets and interview guides aligned with the four commognitive components: word use, visual mediators, narratives, and routines. These instruments were validated by experts in mathematics education. Data analysis followed the stages of collection, reduction, presentation, and conclusion drawing. The findings reveal that high-ability students exhibited more coherent narratives and consistent use of mathematical routines. In contrast, students with moderate and low ability demonstrated fragmented discourse and less effective use of visual mediators and terminology. This study contributes to the understanding of how students' thinking processes vary across ability levels and offers insights into the design of learning tools that support the development of mathematical discourse. Ethical approval was obtained, and all participants provided informed consent in accordance with research ethics standards.

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1. INTRODUCTION

Education plays a crucial role as it serves as a fundamental vehicle for development and knowledge dissemination (Suwartini, 2017). One of the key approaches to developing high-quality human resources

is by enhancing the quality of education (Sudarsana, 2015), particularly through cultivating creative, critical, and logical thinking skills, as well as the ability to collaborate effectively (Suwartini, 2017). However, field observations reveal a significant gap between educational goals and classroom realities. Many students are still trained through rote memorization, with limited exposure to learning strategies that foster engagement and deeper understanding (Rudiansyah et al., 2016). Teachers often rely on monotonous lecture-based methods, which contribute to student fatigue and disengagement (Tarigan et al., 2019).

In response to these challenges, the use of innovative teaching materials and instructional models has become essential to improve the learning process. One such material is the student worksheet, which functions not only as a tool for structured learning but also as a medium to stimulate active participation and teacher-student interaction (Marsa et al., (2016); Mulyani & F (2020). When designed using a problem-based learning (PBL) model, student worksheets can enhance students' problem-solving abilities by placing them in real-life contextual scenarios.

Understanding how students engage with these materials requires a deeper analysis of their thinking processes. This is where the commognitive approach becomes relevant. Commognition—coined by Anna Sfard from the words "communication" and "cognition"—proposes that thinking and communicating are essentially the same process (Sfard, (2016); Zayyadi & Pratiwi, (2022). According to Sfard, thinking can be viewed as an internal dialogue or self-communication.

Commognitive theory identifies four core elements of student discourse by Zayyadi, et al. (2019): (1) word use—how students employ mathematical terminology to express concepts; (2) visual mediators—such as diagrams, symbols, or images used in reasoning; (3) narratives—students' construction of mathematical statements like definitions or theorems; and (4) routines—the recognizable patterns or steps students apply when solving problems (Setyowati et al., 2022). These components are particularly important in a PBL context, as they provide a structured lens through which to analyze students' engagement with complex, real-world tasks.

The use of commognitive analysis in mathematics education allows researchers to understand student thinking not only from their final answers but through how they construct and express their reasoning during problem-solving. As Supardi et al., (2021) assert, this approach can identify difficulties students face at a processual level. In line with this, D. Kim et al (2017) emphasize that commognitive discourse analysis offers valuable insights into the ways students learn and internalize mathematical concepts in classroom settings.

This study focuses specifically on exploring students' commognitive processes in the context of solving problem-based learning student worksheets in Grade XI. The research contributes to the theoretical development of mathematics education by highlighting how commognition can serve as a lens for understanding students' cognitive engagement with problem-based learning tasks. By examining the interplay of words, visual mediators, narratives, and routines in students' discourse, this study addresses a crucial gap in how student thinking is assessed and understood within contemporary classroom practices.

2. METHODS

This study employs a qualitative descriptive research design, which aims to gather information regarding the status of a particular phenomenon, capturing it as it exists at the time of the study. The primary objective of descriptive research is to systematically describe a phenomenon along with its inherent characteristics. This study adopts a qualitative approach and was conducted at SMA Muhammadiyah, with research subjects comprising three students representing high, moderate, and low ability levels. The selection of one student per ability level is justified by the case study design's focus on in-depth, context-rich exploration of individual experiences, which is a core strength of qualitative research. A sample size of three allows for a comparative analysis across diverse levels of ability while maintaining depth and thoroughness in exploring each student's experiences.

The research instruments utilized include: (1) a written test, specifically in the form of a problem-based learning-oriented student worksheet, and (2) a semi-structured interview guide. These instruments were developed by the researcher to ensure the emergence of all commognitive components, namely word use, visual mediators, narratives, and routines. To ensure the validity of these instruments, they were subjected to expert judgment. The validators were two mathematics education specialists and one expert in cognitive development. The rubric used for validation involved assessing the tasks' ability to provoke the four key commognitive components and whether they aligned with the research objectives. Additionally, the worksheet's clarity, mathematical relevance, and capacity to engage students were evaluated.

The data collection techniques employed in this study aim to systematically obtain data to address the research questions. Although the sample consisted of only one student per ability level, this approach is methodologically acceptable in qualitative research as it supports rich, in-depth case-oriented analysis rather than generalization. The data analysis process follows a structured approach, encompassing data collection, data reduction, data presentation, and conclusion drawing.

Sample questions on the worksheet aimed to assess students' mathematical reasoning in problem-solving contexts. For example, one of the problems involved a real-world scenario requiring the application of algebraic reasoning to solve a practical problem, such as determining the cost of materials given a budget constraint. Other problems involved visual aids like diagrams to assess spatial reasoning and interpretations of mathematical relationships. Prior to data collection, ethical approval was obtained, and all participants provided informed consent to ensure adherence to ethical research standards.

3. FINDINGS AND DISCUSSION

Findings

The data for this study were obtained from the final examination scores of the odd semester in the mathematics subject for Grade XI IPA 2, involving a total of 19 students. The researcher categorized students into three ability levels: high, moderate, and low. This classification was conducted based on the categorization framework proposed by Azwar; Widhiarso (2014). The ability levels were determined using the following formulas:

Table 1. Categorization of Student Abilities Based on Scores.

Ability Category	Formula	Score Range
High	$X > M + 1SD$	$X > 63.8$
Moderate	$M + 1SD < X \leq M - 1SD$	$35.06 \leq X < 63.82$
Low	$X \leq M - 1SD$	$X \leq 35.06$

High-Ability Subject (NSU)

Word Use

NSU used both mathematical (e.g., area, maximum, perimeter, derivative) and everyday terms (e.g., time). This shows conceptual flexibility and fluency in math discourse.

Visual Mediators

NSU employed iconic (e.g., sketches of land) and symbolic mediators (e.g., K, P, l, L') throughout all problem-solving steps. Absence of concrete mediators was acknowledged due to unavailability of tools. This indicates visual literacy and symbol manipulation skills.

Routine

NSU followed PBL steps systematically, applying formulas and derivatives (e.g., L' = 0) to find a maximum area of 5,000 m². This demonstrates a structured problem-solving strategy.

Narrative

NSU accurately recalled and applied information, reasoning logically through each worksheet step and interview. The narrative confirms a thorough understanding and metacognitive awareness.

Moderate-Ability Subject (PM)

Word Use

PM used correct terms like area, side, and length, though sometimes misaligned them with instructions (e.g., giving known information instead of formulating a problem). This suggests partial understanding.

Visual Mediators

Both iconic and symbolic mediators were used (e.g., sketches, symbols p, l, K, L'). No concrete media was used, with time constraints cited. Indicates an ability to represent math visually, though possibly under pressure.

Routine

PM completed all steps, derived formulas, and calculated a 3,200 m² area using L' = 0. This shows procedural understanding but with minor conceptual slips.

Narrative

PM followed instructions but often provided information in the wrong sections. Still, the process of drawing and formula use shows emerging understanding.

Low-Ability Subject (AR)

Word Use

AR used relevant terms (e.g., rectangle, length, perimeter), with evidence of dual-meaning terms (e.g., kali). Some responses lacked precision, reflecting limited mathematical vocabulary usage.

Visual Mediators

AR used sketches and symbolic mediators (e.g., P, l, L, L') in most steps but did not use concrete visualizations. Demonstrates basic visual representation ability, though with limited elaboration.

Routine

AR followed the worksheet structure but struggled with correct formula application—especially with L' = 0—leading to incorrect conclusions.

Narrative

AR's narrative reflects minimal understanding. Though the drawing and data collection were done, key conceptual errors remained unresolved.

Table 2. Subject Comparison by Commognitive Component

Component	High (NSU)	Moderate (PM)	Low (AR)
Word Use	Fluent with math and everyday terms	Adequate, occasional confusion	Basic, imprecise
Visual Mediator	Iconic & symbolic used effectively	Used, but no concrete visuals	Basic use, no concrete visuals
Routine	Systematic and correct application	Follows structure, minor errors	Follows steps, conceptual errors

Narrative	Logical, detailed, and accurate	Adequate but misplaced responses	Minimal and sometimes incorrect

Discussion

Differences in Word Use Across Ability Levels

High-Ability Student (NSU Subject)

The NSU subject demonstrated comprehensive use of all four commognitive components, particularly excelling in word use. The subject employed seven mathematical and one non-mathematical term throughout the problem-solving process. Word use was consistent and precise across various stages, including problem formulation, analysis, hypothesis development, data collection, and testing (Zayyadi et al., 2023). This precision in mathematical language aligns with the formal operational stage of Piaget's theory, where abstract and systematic thinking becomes prominent.

Moderate-Ability Student (Subject PM)

Subject PM also employed both mathematical and non-mathematical terms, but primarily during problem comprehension. The student faced challenges in problem formulation and analysis, indicating gaps in applying mathematical language precisely (Zayyadi et al., 2023). This suggests that while PM may be transitioning into formal operational thinking, there remains reliance on concrete strategies.

Limited-Ability Student (Subject AR)

Subject AR used mathematical language beginning at the problem formulation stage, mirroring NSU and PM in the range of terms used. However, the depth of conceptual understanding in applying this terminology was limited, as seen in the inaccurate responses within problem formulation and final answers. This supports Vygotsky's notion that language use without internalized meaning may not reflect actual cognitive development, emphasizing the need for scaffolded instruction.

Visual Mediators and Abstract Thinking

Across all subjects, visual mediators (iconic and symbolic) were utilized effectively. All three subjects drew land and fence diagrams, incorporating symbols such as p, l, and k. However, none used concrete visual media, which aligns with Piaget's formal operational stage, where individuals are expected to think abstractly without relying on tangible objects (Marinda, 2020).

High-Ability Student (NSU Subject)

The NSU subject integrated visual mediators with strong abstract reasoning, demonstrating how symbolic elements enhanced conceptual understanding. This aligns with Sfard's view of visual mediators as integral tools for internal communication of mathematical concepts.

Moderate-Ability Student (Subject PM)

Subject PM mirrored NSU's use of diagrams but showed less conceptual clarity. Although structurally accurate, their diagrams did not fully support the reasoning process, suggesting limitations in transforming visual tools into deeper understanding.

Limited-Ability Student (Subject AR)

Subject AR's visual representation was similar to the others but lacked conceptual integration, contributing to misinterpretations in the final solution. The student may have entered the formal operational stage but struggled to use visual mediators as vehicles for logical abstraction.

Routine Use and Flexibility

All students demonstrated ritual, explorative, and applicable routines by using formulas for perimeter, area, and maximum area via derivatives. This suggests that procedural knowledge was present across ability levels, but the depth of understanding varied.

High-Ability Student (NSU Subject)

NSU excelled in routine flexibility and explorative thinking. They modified perimeter formulas and logically sequenced steps, reflecting high procedural fluency and supporting (Mudaly & Mpofu, (2019) observation on adaptable routine use. Unlike the typical ritual routines seen in many students, NSU shifted toward personalized strategies—demonstrating a novel contribution to commognitive literature.

Moderate-Ability Student (Subject PM)

PM displayed some routine flexibility (e.g., converting formulas), but execution was inconsistent. The conceptual gaps suggest a need for guided discovery learning to foster more autonomous thinking.

Limited-Ability Student (Subject AR)

Subject AR showed formula adaptation but struggled in derivative application—indicating an incomplete grasp of higher-order routines. This highlights the need for more explicit modeling and metacognitive prompting in classroom teaching.

Narrative Construction and Cognitive Engagement

High-Ability Student (NSU Subject)

NSU constructed a coherent narrative, personally engaging with each problem step using first-person pronouns, which signals metacognitive awareness (Rossydha et al., 2021). This reflects a high level of internal dialogue, consistent with Sfard's concept of thinking as communication with the self.

Moderate-Ability Student (Subject PM)

PM's narrative improved over time. While initial steps lacked clarity, retrospective explanations in interviews showed emerging cognitive control and reflection—suggesting growth through social interaction, in line with Vygotsky's sociocultural theory.

Limited-Ability Student (Subject AR)

AR followed the worksheet structure but showed incomplete narrative formation. The final answer was incorrect, yet interview responses revealed emerging reflection skills. This underscores the potential of narrative scaffolding in enhancing self-explanation and conceptual clarity.

Critical Comparison with Literature

The findings affirm Setyowati et al., (2022),, who reported that all commognitive components are present in students with visual learning styles. However, this study contributes novel insights by showing that ability level influences not just the presence, but the *depth* and *accuracy* of these components. Unlike prior studies which generalized commognitive use, this research demonstrates that high-ability students engage more flexibly and metacognitively with mathematical discourse.

Furthermore, the ritualized routines discussed by (Mudaly & Mpofu, (2019) were evident across all participants, but only NSU transitioned toward exploratory and adaptive routine use—a deviation from previous findings. This supports the idea that routine development is mediated by both cognitive ability and instructional design.

Theoretical Contribution

This study extends Sfard's commognitive theory by applying it within the context of Indonesian problem-based learning (PBL) classrooms, offering empirical evidence on how commognitive elements manifest differently across student ability levels. Importantly, it highlights that commognition is not solely an individual process, but is shaped by instructional models, cultural norms, and teacher discourse.

By situating this analysis within a PBL framework, the study shows how structured yet open-ended learning can surface diverse depths of mathematical discourse. These findings underscore the importance of designing learning environments that allow for differentiated student expression, and encourage teachers to attend not only to student answers, but also to the how and why behind their reasoning. This has practical implications for lesson planning, formative assessment, and the integration of reflective discourse in mathematics classrooms.

4. CONCLUSION

Based on the presentation of research findings and discussion, it can be concluded that the high-ability subject demonstrated all four components of commognitive theory within the context of the problem-based learning Student Worksheets. The subject employed word use that included both mathematical and non-mathematical terms. The visual mediators used were both iconic and symbolic in nature. In terms of routines, the subject applied formulas related to the area and perimeter of rectangles, as well as the formula for maximum area in the context of derivatives. The subject also engaged in both ritual and exploratory routines. The narrative component was reflected in the subject's demonstrated understanding of how to complete the Student Worksheets and apply the appropriate procedures and formulas.

The medium-ability subject demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. However, in the step of problem formulation, the subject included some information that overlapped with the data collection phase, resulting in limited written input during the data-gathering section. Consequently, the word use component appeared only to a minimal extent. The subject used both mathematical and non-mathematical terms. The visual mediators employed by the medium-ability subject were similar to those used by the high-ability subject, consisting of both iconic and symbolic representations. Regarding routines, the subject applied formulas for the area and perimeter of rectangles, as well as for determining the maximum area using derivatives. Similar to the high-ability subject, the medium-ability subject also demonstrated both ritual and exploratory routines. The narrative component was reflected in the subject's understanding of how to complete the problem-based learning Student Worksheets and apply the relevant procedures and formulas.

The low-ability subject also demonstrated all four components of commognitive theory within the problem-based learning Student Worksheets. The subject used both mathematical and non-mathematical terminology as part of the word use component. The visual mediators employed were iconic and symbolic, similar to those used by the high-ability subject. In terms of routines, the subject engaged in both ritual and exploratory routines. However, the subject provided an incorrect answer in the final step due to a misunderstanding of the concept of derivative applications. The narrative component was evident in the subject's understanding of several procedural steps within the Student Worksheets and a partial grasp of the formulas and procedures used.

In summary, all three subjects—high, medium, and low ability—exhibited the four components of commognitive theory through their engagement with the problem-based learning Student Worksheets. This indicates the effectiveness of problem-based learning in fostering mathematical communication and cognitive development across diverse student abilities.

Theoretical Implications

These findings contribute to the extension of Sfard's commognitive theory by demonstrating that students of varying abilities can exhibit the theory's four components—word use, visual mediators, routines, and narratives—within a structured PBL environment. The study provides empirical support for the commognitive approach in exploring how mathematical thinking is constructed and communicated during problem-solving, especially in secondary-level mathematics education.

Practical Implications

From a pedagogical perspective, the study suggests that problem-based learning materials should be designed intentionally to activate all four commognitive components. Teachers and curriculum developers should incorporate scaffolding strategies—especially for lower-ability students—to support more accurate application of mathematical concepts, particularly in abstract topics such as derivatives. Emphasizing clarity in student narratives and symbolic representations may also improve conceptual integration and reduce misconceptions.

Recommendations for Educators and Researchers

Educators are encouraged to integrate commognitive elements explicitly into classroom instruction and assessment, using PBL worksheets as tools to reveal and develop student thinking. Researchers should investigate how variations in worksheet design or teacher facilitation influence each commognitive component. Future studies could also explore the long-term development of student discourse through repeated PBL exposure, and examine teacher-student interactions as key drivers of commognitive growth in classroom settings.

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